Open Questions in Open Systems

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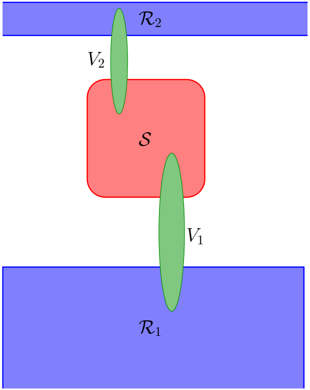
Open quantum systems



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- Mean entropy production rate

$$\mathrm{Ep}(\omega^+) = -\lim_{t\to\infty}\frac{1}{t}\mathrm{Ent}(\omega\circ\tau^t|\omega).$$

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$$w - \lim_{t\to\infty} \mathrm{e}^{t\mathcal{L}^*}(\omega) = \rho.$$

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  generates a unitary group U^t on $\mathcal{Z}$.
- Associated Markovian semigroup

$$\mathrm{e}^{t\mathcal{L}}(A) = \langle \Omega | U^t (A \otimes I) U^{t*} | \Omega \rangle$$

  is generated by

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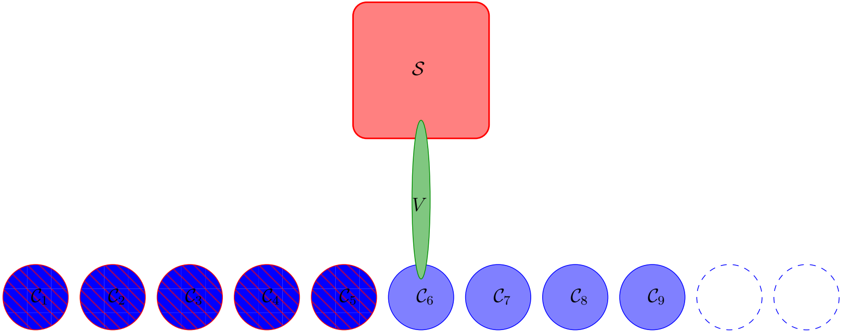
Repeated interaction models



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- Repeated interaction [Bruneau-Joye-Merkli, Bruneau-P].
- Current fluctuations [Avron-Bachmann-Graf-Klich, De Roeck-Dereziński-Maes].

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- Generalized weak coupling limit [Taj].

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- Fluctuation-dissipation near equilibrium [Jakšić-Ogata-Pautrat-P].

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- A starting point may be a more clever choice of the cyclic vector in the standard representation of $\mathfrak{A}$.

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- Exact solution, via Bethe Ansatz ?

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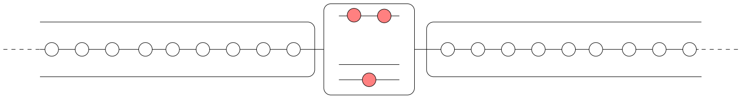
Example (The Anderson model)



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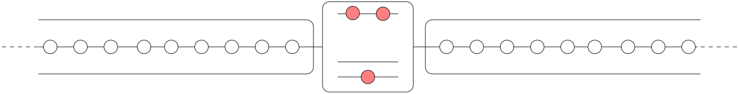

$$H_L = -t \sum_{x<0,\sigma} a^*_\sigma(x) a_\sigma(x-1) + a^*_\sigma(x-1) a_\sigma(x)$$

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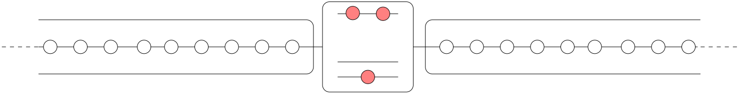

$$H_R = -t \sum_{x>0,\sigma} a^*_\sigma(x) a_\sigma(x+1) + a^*_\sigma(x+1) a_\sigma(x)$$

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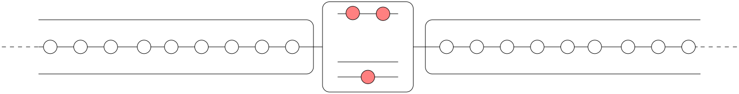

$$H_D = -\sum_{\sigma} \varepsilon_0 a^*_\sigma(0) a_\sigma(0) + U a^*_-(0) a_-(0) a^*_+(0) a_+(0)$$

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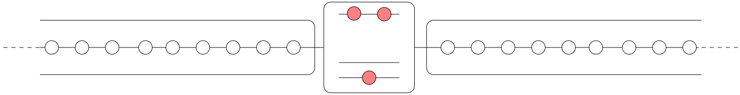

$$H = H_L + H_R + H_D + \lambda \sum_{x\in\{\pm 1\},\sigma} a^*_\sigma(x)a_\sigma(0) + a^*_\sigma(0)a_\sigma(x)$$

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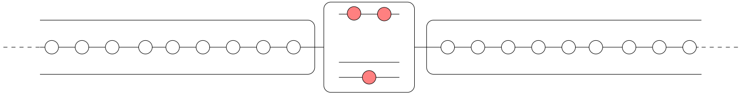

The Kondo regime

$$U \gg \lambda \simeq t$$

is non-perturbative.

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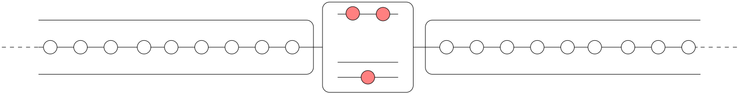

Is there a "non-linear" Landauer-Büttiker formula ?

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  - in the C^*-scattering approach, the Dyson expansion can not be controlled uniformly in the "dimension" of the perturbation.

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- Quantum Brownian motion.

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- Apply the RW approximation to matter-radiation problems and do the relevant spectral analysis (generalization of Friedrichs type models).

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