

On the Keldysh formalism applied to mesoscopic quantum transport

Horia D. Cornean

Department of Mathematical Sciences, Aalborg University, Fredrik Bajers Vej 7G, 9220 Aalborg, Denmark; e-mail: cornean@math.aau.dk

Grenoble, November 30, 2010

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- The current formula: $I_\alpha(t) = 2\text{Re}\{V_\alpha(t)G_{m_\alpha, 0_\alpha}^<(t, t)\}$

The noninteracting case

- $G_{x,x'}^<(t,t') = \langle a_H^*(|x'\rangle, t') a_H(|x\rangle, t) \rangle_{\text{ref}} = \langle a^*(u^*(t', t_0)|x'\rangle) a(u^*(t, t_0)|x\rangle) \rangle_{\text{ref}},$

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The Keldysh equation

- Duhamel: $u(t, t_0) = e^{-i(t-t_0)h_0} - i\int_{t_0}^t u(t, s)h_T(s)e^{-i(s-t_0)h_0}ds$

The noninteracting case

- $G_{x,x'}^<(t,t') = i\langle a_H^*(|x'\rangle, t')a_H(|x\rangle, t)\rangle_{\text{ref}} = i\langle a^*(u^*(t', t_0)|x'\rangle)a(u^*(t, t_0)|x\rangle)\rangle_{\text{ref}},$
- or: $G_{x,x'}^<(t,t') = i\sum_\gamma \langle x, u(t, t_0) f_\gamma(h_L) P_\gamma u^*(t', t_0) x' \rangle + i\sum_\lambda n_\lambda \langle \phi_\lambda, u^*(t', t_0) x' \rangle \langle u^*(t, t_0) x, \phi_\lambda \rangle$

The retarded Keldysh function

- $G_{x,x'}^R(t,t') := -i\theta(t-t')\langle [a_H^*(|x'\rangle, t'), a_H(|x\rangle, t)]_+\rangle_{\text{ref}}$
- or: $G_{x,x'}^R(t,t') = -i\theta(t-t')\langle x, u(t, t') x' \rangle$

The self-energy of the leads

- $\Sigma_\gamma^<(s, s') := V_\gamma(s)V_\gamma(s')\langle 0_\gamma, f_\gamma(h_L^{(\gamma)})e^{i(s-s')h_L^{(\gamma)}}0_\gamma \rangle$

The Keldysh equation

- Duhamel: $u(t, t_0) = e^{-i(t-t_0)h_0} - i \int_{t_0}^t u(t, s) h_T(s) e^{-i(s-t_0)h_0} ds$
- Keldysh:

$$G_{m,m'}^<(t,t') = \sum_\gamma \int_{t_0}^t ds G_{m,m_\gamma}^R(t,s) \int_{t_0}^{t'} ds' \Sigma_\gamma^<(s,s') G_{m_\gamma,m'}^A(s',t')$$

$$+ i \sum_\lambda n_\lambda \langle \phi_\lambda, u^*(t', t_0) m' \rangle \langle u^*(t, t_0) m, \phi_\lambda \rangle$$

The Jauho-Meir formula

The Jauho-Meir formula

$$I_\alpha(t) = 2\text{Re}\{V_\alpha(t)G_{m_\alpha,0_\alpha}^<(t,t)\}$$

The Jauho-Meir formula

$$\begin{aligned} I_\alpha(t) &= 2\text{Re}\{V_\alpha(t)G_{m_\alpha,0_\alpha}^<(t,t)\} \\ &= \int_{t_0}^t ds \int_{-2}^2 dE \sqrt{4-E^2} V_\alpha(s) V_\alpha(t) e^{-i(s-t)E} \{G_{m_\alpha,m_\alpha}^<(t,s) + f_\alpha(E) G_{m_\alpha,m_\alpha}^R(t,s)\} \end{aligned}$$

The Jauho-Meir formula

$$\begin{aligned}
I_\alpha(t) &= 2\text{Re}\{V_\alpha(t)G_{m_\alpha,0_\alpha}^<(t,t)\} \\
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&= \int_0^{t-t_0} ds V_\alpha(t-s) V_\alpha(t) \{G_{m_\alpha,m_\alpha}^<(t,t-s) \int_{-2}^2 dE \sqrt{4-E^2} e^{isE} \\
&\quad + G_{m_\alpha,m_\alpha}^R(t,t-s) \int_{-2}^2 dE \sqrt{4-E^2} e^{isE} f_\alpha(E)\}
\end{aligned}$$

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&= \int_0^{t-t_0} ds V_\alpha(t-s) V_\alpha(t) \{G_{m_\alpha,m_\alpha}^<(t,t-s) \int_{-2}^2 dE \sqrt{4-E^2} e^{isE} \\
&\quad + G_{m_\alpha,m_\alpha}^R(t,t-s) \int_{-2}^2 dE \sqrt{4-E^2} e^{isE} f_\alpha(E)\}
\end{aligned}$$

Steady state?

The Jauho-Meir formula

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&= \int_0^{t-t_0} ds V_\alpha(t-s) V_\alpha(t) \{G_{m_\alpha,m_\alpha}^<(t,t-s) \int_{-2}^2 dE \sqrt{4-E^2} e^{isE} \\
&\quad + G_{m_\alpha,m_\alpha}^R(t,t-s) \int_{-2}^2 dE \sqrt{4-E^2} e^{isE} f_\alpha(E)\}
\end{aligned}$$

Steady state?

$$G_{m_\alpha,m_\alpha}^<(t,t-s) = i \sum_\gamma \langle m_\alpha, u(t,t_0) f_\gamma(h_L) P_\gamma u^*(t-s,t_0) m_\alpha \rangle$$

The Jauho-Meir formula

$$\begin{aligned}
I_\alpha(t) &= 2\text{Re}\{V_\alpha(t)G_{m_\alpha,0_\alpha}^<(t,t)\} \\
&= \int_{t_0}^t ds \int_{-2}^2 dE \sqrt{4-E^2} V_\alpha(s) V_\alpha(t) e^{-i(s-t)E} \{G_{m_\alpha,m_\alpha}^<(t,s) + f_\alpha(E) G_{m_\alpha,m_\alpha}^R(t,s)\} \\
&= \int_0^{t-t_0} ds V_\alpha(t-s) V_\alpha(t) \{G_{m_\alpha,m_\alpha}^<(t,t-s) \int_{-2}^2 dE \sqrt{4-E^2} e^{isE} \\
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- $u(t,t_0) = e^{-i(t-t_1)h} u(t_1,t_0)$ and $u(t-s,t_0) = e^{-i(t-s-t_1)h} u(t_1,t_0)$

The Jauho-Meir formula

$$\begin{aligned}
I_\alpha(t) &= 2\text{Re}\{V_\alpha(t)G_{m_\alpha,0_\alpha}^<(t,t)\} \\
&= \int_{t_0}^t ds \int_{-2}^2 dE \sqrt{4-E^2} V_\alpha(s) V_\alpha(t) e^{-i(s-t)E} \{G_{m_\alpha,m_\alpha}^<(t,s) + f_\alpha(E) G_{m_\alpha,m_\alpha}^R(t,s)\} \\
&= \int_0^{t-t_0} ds V_\alpha(t-s) V_\alpha(t) \{G_{m_\alpha,m_\alpha}^<(t,t-s) \int_{-2}^2 dE \sqrt{4-E^2} e^{isE} \\
&\quad + G_{m_\alpha,m_\alpha}^R(t,t-s) \int_{-2}^2 dE \sqrt{4-E^2} e^{isE} f_\alpha(E)\}
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- $e^{i(t_1-t_0)h_0} u(t_1,t_0) - \text{Id}$ and $u^*(t_1,t_0) e^{-i(t_1-t_0)h_0} - \text{Id}$ are compact

The Jauho-Meir formula

$$\begin{aligned}
I_\alpha(t) &= 2\text{Re}\{V_\alpha(t)G_{m_\alpha,0_\alpha}^<(t,t)\} \\
&= \int_{t_0}^t ds \int_{-2}^2 dE \sqrt{4-E^2} V_\alpha(s) V_\alpha(t) e^{-i(s-t)E} \{G_{m_\alpha,m_\alpha}^<(t,s) + f_\alpha(E) G_{m_\alpha,m_\alpha}^R(t,s)\} \\
&= \int_0^{t-t_0} ds V_\alpha(t-s) V_\alpha(t) \{G_{m_\alpha,m_\alpha}^<(t,t-s) \int_{-2}^2 dE \sqrt{4-E^2} e^{isE} \\
&\quad + G_{m_\alpha,m_\alpha}^R(t,t-s) \int_{-2}^2 dE \sqrt{4-E^2} e^{isE} f_\alpha(E)\}
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- $u(t,t_0) = e^{-i(t-t_1)h} u(t_1,t_0)$ and $u(t-s,t_0) = e^{-i(t-s-t_1)h} u(t_1,t_0)$
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$$G_{m_\alpha,m_\alpha}^<(t,t-s) = i \sum_\gamma \langle m_\alpha, e^{-i(t-t_1)h} u(t_1,t_0) f_\gamma(h_L) P_\gamma u^*(t_1,t_0) e^{i(t-s-t_1)h} m_\alpha \rangle$$

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&= \int_0^{t-t_0} ds V_\alpha(t-s) V_\alpha(t) \{G_{m_\alpha,m_\alpha}^<(t,t-s) \int_{-2}^2 dE \sqrt{4-E^2} e^{isE} \\
&\quad + G_{m_\alpha,m_\alpha}^R(t,t-s) \int_{-2}^2 dE \sqrt{4-E^2} e^{isE} f_\alpha(E)\}
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G_{m_\alpha,m_\alpha}^<(t,t-s) &= i \sum_\gamma \langle m_\alpha, e^{-i(t-t_1)h} u(t_1,t_0) f_\gamma(h_L) P_\gamma u^*(t_1,t_0) e^{i(t-s-t_1)h} m_\alpha \rangle \\
&\approx i \sum_\gamma \langle m_\alpha, e^{-i(t-t_1)h} f_\gamma(h_L) P_\gamma e^{i(t-s-t_1)h} m_\alpha \rangle
\end{aligned}$$

The Jauho-Meir formula

$$\begin{aligned}
I_\alpha(t) &= 2\text{Re}\{V_\alpha(t)G_{m_\alpha,0_\alpha}^<(t,t)\} \\
&= \int_{t_0}^t ds \int_{-2}^2 dE \sqrt{4-E^2} V_\alpha(s) V_\alpha(t) e^{-i(s-t)E} \{G_{m_\alpha,m_\alpha}^<(t,s) + f_\alpha(E) G_{m_\alpha,m_\alpha}^R(t,s)\} \\
&= \int_0^{t-t_0} ds V_\alpha(t-s) V_\alpha(t) \{G_{m_\alpha,m_\alpha}^<(t,t-s) \int_{-2}^2 dE \sqrt{4-E^2} e^{isE} \\
&\quad + G_{m_\alpha,m_\alpha}^R(t,t-s) \int_{-2}^2 dE \sqrt{4-E^2} e^{isE} f_\alpha(E)\}
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- $e^{i(t_1-t_0)h_0} u(t_1,t_0) - \text{Id}$ and $u^*(t_1,t_0) e^{-i(t_1-t_0)h_0} - \text{Id}$ are compact

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G_{m_\alpha,m_\alpha}^<(t,t-s) &= i \sum_\gamma \langle m_\alpha, e^{-i(t-t_1)h} u(t_1,t_0) f_\gamma(h_L) P_\gamma u^*(t_1,t_0) e^{i(t-s-t_1)h} m_\alpha \rangle \\
&\approx i \sum_\gamma \langle m_\alpha, e^{-i(t-t_1)h} f_\gamma(h_L) P_\gamma e^{i(t-s-t_1)h} m_\alpha \rangle
\end{aligned}$$

$$\lim_{t \rightarrow \infty} G_{m_\alpha,m_\alpha}^<(t,t-s) = i \sum_\gamma \langle m_\alpha, \omega_- f_\gamma(h_L) e^{-is h_L} P_\gamma \omega_-^* m_\alpha \rangle$$

The Jauho-Meir formula

$$\begin{aligned}
I_\alpha(t) &= 2\text{Re}\{V_\alpha(t)G_{m_\alpha,0_\alpha}^<(t,t)\} \\
&= \int_{t_0}^t ds \int_{-2}^2 dE \sqrt{4-E^2} V_\alpha(s) V_\alpha(t) e^{-i(s-t)E} \{G_{m_\alpha,m_\alpha}^<(t,s) + f_\alpha(E) G_{m_\alpha,m_\alpha}^R(t,s)\} \\
&= \int_0^{t-t_0} ds V_\alpha(t-s) V_\alpha(t) \{G_{m_\alpha,m_\alpha}^<(t,t-s) \int_{-2}^2 dE \sqrt{4-E^2} e^{isE} \\
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Steady state?

$$G_{m_\alpha,m_\alpha}^<(t,t-s) = i \sum_\gamma \langle m_\alpha, u(t,t_0) f_\gamma(h_L) P_\gamma u^*(t-s,t_0) m_\alpha \rangle$$

- $u(t,t_0) = e^{-i(t-t_1)h} u(t_1,t_0)$ and $u(t-s,t_0) = e^{-i(t-s-t_1)h} u(t_1,t_0)$
- $e^{i(t_1-t_0)h_0} u(t_1,t_0) - \text{Id}$ and $u^*(t_1,t_0) e^{-i(t_1-t_0)h_0} - \text{Id}$ are compact

$$\begin{aligned}
G_{m_\alpha,m_\alpha}^<(t,t-s) &= i \sum_\gamma \langle m_\alpha, e^{-i(t-t_1)h} u(t_1,t_0) f_\gamma(h_L) P_\gamma u^*(t_1,t_0) e^{i(t-s-t_1)h} m_\alpha \rangle \\
&\approx i \sum_\gamma \langle m_\alpha, e^{-i(t-t_1)h} f_\gamma(h_L) P_\gamma e^{i(t-s-t_1)h} m_\alpha \rangle
\end{aligned}$$

$$\lim_{t \rightarrow \infty} G_{m_\alpha,m_\alpha}^<(t,t-s) = i \sum_\gamma \langle m_\alpha, \omega_- f_\gamma(h_L) e^{-is h_L} P_\gamma \omega_-^* m_\alpha \rangle$$

$$\begin{aligned}
I_\alpha(\infty) &= i \int_0^\infty ds \sum_\gamma \langle m_\alpha, \omega_- f_\gamma(h_L) e^{-is h_L} P_\gamma \omega_-^* m_\alpha \rangle \int_{-2}^2 dE \sqrt{4-E^2} e^{isE} \\
&\quad - i \int_0^\infty ds \langle m_\alpha, e^{is h} m_\alpha \rangle \int_{-2}^2 dE f_\alpha(E) \sqrt{4-E^2} e^{isE}.
\end{aligned}$$