Engineering inverse power law decoherence of a qubit

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Strategy

Correct Theoretical Physics Strategy

At Mathematics Conference speak about Physics!
At Physics Conference speak about Mathematics!

Strategy

Correct Theoretical Physics Strategy

At Mathematics Conference speak about Physics!
At Physics Conference speak about Mathematics!

Dangerous strategy

At Mathematics Conference speak about Mathematics!

Outline

Outline

Outline

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Situation and strategy

Situation

- Analytical Exponential-like Decoherence processes for Lorentzian type distribution of field modes in Jaynes-Cummings model
- Oscillating decay and trapping for distribution of field modes with photonic band gap (PBG) edge near the resonant frequency of the two-level system

Strategy

- Delay the Decoherence process by engineering the reservoirs of field modes
- Search for inverse power laws in the exact dynamics

The Jaynes-Cummings model

The Hamiltonian of the whole system:

$$H = H_S + H_E + H_I, \qquad \hbar = 1$$

$$H_S = \omega_0 \sigma_+ \sigma_-, \;\; H_E = \sum_{k=1}^{\infty} \omega_k a_k^\dagger a_k, \;\; H_I = \sum_{k=1}^{\infty} \left(g_k \sigma_+ \otimes a_k + g_k^* \sigma_- \otimes a_k^\dagger \right)$$

The operators acting on the Hilbert space of the qubit:

$$\sigma_+ |0\rangle = |1\rangle, \quad \sigma_+ |1\rangle = 0, \quad \sigma_- = \sigma_+^\dagger$$

The operators acting on the Hilbert space of the field modes:

$$a_k^\dagger | \cdots, n_k, \cdots \rangle_E = \sqrt{n_k + 1} \, | \cdots, n_k + 1, \cdots \rangle_E$$

$$N = \sigma_+ \sigma_- + \sum_{k=1}^{\infty} a_k^\dagger a_k, \quad [H, N] = [H_I, N] = 0$$

Initial condition and time evolution

Initial unentangled condition between the qubit and the vacuum state of the external environment:

$$|\Psi(0)\rangle = (c_0|0\rangle + c_1(0)|1\rangle) \otimes |0\rangle_E$$

Exact time evolution

$$|\Psi(t)\rangle = c_0\,|0\rangle \otimes |0\rangle_E + c_1(t)\,|1\rangle \otimes |0\rangle_E + \sum_{k=1}^{\infty} d_k(t)\,|0\rangle \otimes |k\rangle_E$$

where

$$|k\rangle_E = a_k^\dagger |0\rangle_E, \qquad k = 1, 2, \cdots$$

The equations of the exact dynamics: Ansatz

Interaction picture

$$
\begin{aligned}
|\Psi(t)\rangle_I &= e^{\imath(H_S+H_E)t}|\Psi(t)\rangle \\
&= c_0|0\rangle \otimes |0\rangle_E + C_1(t)|1\rangle \otimes |0\rangle_E + \sum_{k=1}^{\infty} \Lambda_k(t)\,|0\rangle \otimes |k\rangle_E
\end{aligned}
$$

where

$$\Lambda_k(t) = e^{\imath\omega_k t}\, d_k(t), \quad k = 1, 2, \ldots$$

The equations of the exact dynamics: Convolution equation

Equations for the coefficients:

$$\begin{aligned}\dot{C}_1(t) &= -\imath \sum_{k=1}^{\infty} g_k e^{\imath(\omega_0-\omega_k)t} \Lambda_k(t), \\ \dot{\Lambda}_k(t) &= -\imath\, g_k^* \, e^{-\imath(\omega_0-\omega_k)t} \, C_1(t)\end{aligned}$$

Closed equation for $C_1(t)$

$$\dot{C}_1(t) = -\left(f * C_1\right)(t),$$

where two-point correlation function of the reservoir of field modes

$$f\left(t-t'\right) = \sum_{k=1}^{\infty} |g_k|^2 \, e^{-\imath(\omega_k-\omega_0)(t-t')}$$

The correlation function and the spectral density

Two-point correlation function of the reservoir of field modes

$$f\left(t-t'\right)=\sum_{k=1}^{\infty}\left|g_k\right|^2 e^{-\imath(\omega_k-\omega_0)(t-t')}$$

For a continuous distribution of modes $\eta(\omega)$

$$f\left(\tau\right)=\int_0^{\infty} J\left(\omega\right) e^{-\imath(\omega-\omega_0)\tau} d\omega,$$

where

- spectral density function

$$J\left(\omega\right)=\eta\left(\omega\right)\left|g\left(\omega\right)\right|^2$$

- frequency dependent coupling constant $g(\omega)$

Reduced density matrix

By tracing over the degrees of freedom of the reservoir:

$$\begin{aligned} \rho_{1,1}(t) &= = 1 - \rho_{0,0} = \rho_{1,1}(0)\,|G(t)|^2\,, \\ \rho_{1,0}(t) &= \rho_{0,1}^*(t) = \rho_{1,0}(0)\,e^{-\imath\omega_0 t}\,G(t) \end{aligned}$$

The term $G(t)$ fulfills

$$\dot{G}(t) = -\left(f * G\right)(t),$$

with

$$G(0) = 1$$

The Lorentzian spectral density and Exponential-like relaxations (1)

Garraway model (Phys Rev A55 (1997) 2290):

- Lorentzian spectral density function

$$\tilde{J}_L(\omega) = \frac{1}{2\pi} \frac{\gamma\lambda^2}{(\omega - \omega_0)^2 + \lambda^2}$$

- Reservoir correlation function:

$$\tilde{f}_L(\tau) = \int_{-\infty}^{\infty} \tilde{J}_L(\omega)\, e^{-\imath(\omega-\omega_0)\tau} d\omega = \frac{\gamma\lambda}{2}\, e^{-\lambda|\tau|}$$

where

- $\lambda > 0$: spectral width of the coupling
- $\gamma > 0$: relaxation rate

The Lorentzian spectral density and Exponential-like relaxations (2)

The exact dynamics of the qubit:

$$\rho_{1,1}(t) = 1 - \rho_{0,0}(t) = \rho_{1,1}(0)\,|G_L(t)|^2$$
$$\rho_{1,0}(t) = \rho^*_{0,1}(t) = \rho_{1,0}(0)\,e^{-\imath\omega_0 t}\,G_L(t)$$

The weak and strong coupling regimes:

$$G_L(t) = e^{-\lambda t/2}\left(\cosh\left(\frac{d}{2}\,t\right) + \frac{\lambda}{d}\sinh\left(\frac{d}{2}\,t\right)\right), \qquad \lambda > 2\gamma$$
$$G_L(t) = e^{-\lambda t/2}\left(\cos\left(\frac{\hat{d}}{2}\,t\right) + \frac{\lambda}{\hat{d}}\sin\left(\frac{\hat{d}}{2}\,t\right)\right), \qquad \lambda < 2\gamma$$

$$\hat{d} = \sqrt{2\gamma\lambda - \lambda^2}, \qquad d = \sqrt{\lambda^2 - 2\gamma\lambda}$$

Lorentzian type spectral densities

Other known solutions for:

$$\tilde{J}_{L_+}(\omega) = \frac{W_1\,\Gamma_1}{\left(\omega-\omega_r^{(1)}\right)^2+(\Gamma_1/2)^2}+\frac{W_2\,\Gamma_2}{\left(\omega-\omega_r^{(2)}\right)^2+(\Gamma_2/2)^2}$$

$$\tilde{J}_{L'}(\omega) = \frac{4\Gamma^3/\sqrt{2}}{(\omega-\omega_r)^4+\Gamma^4}$$

$$\tilde{J}_{L_-}(\omega) = \frac{W_1\,\Gamma_1}{(\omega-\omega_r)^2+(\Gamma_1/2)^2}-\frac{W_2\,\Gamma_2}{(\omega-\omega_r)^2+(\Gamma_2/2)^2},\ (PBG)$$

(Exponential-like decay and trapping)

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- B.M. Garraway and P.L. Knight . Phys. Rev. A **54** (1996) 3592-3602
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- S. John and T. Quang. Phys. Rev. A **50** (1994) 1764 - 1769

The Fox H-function

Very general function defined by:

$$H_{p,q}^{m,n}\left[z\left|\begin{array}{c}(a_1,\alpha_1),\ldots,(a_p,\alpha_p)\\(b_1,\beta_1),\ldots,(b_q,\beta_q)\end{array}\right.\right]$$
$$=\frac{1}{2\pi\imath}\int_{\mathcal{C}}\frac{\Pi_{j=1}^{m}\Gamma\left(b_j+\beta_j s\right)\Pi_{m=1}^{n}\Gamma\left(1-a_l-\alpha_l s\right)z^{-s}}{\Pi_{l=n+1}^{p}\Gamma\left(a_l+\alpha_l s\right)\Pi_{j=m+1}^{q}\Gamma\left(1-b_j-\beta_j s\right)}\,ds$$

- $0\leq m\leq q$, $0\leq n\leq p$
- α_j, $\beta_j>0$; a_j, b_j complex numbers such that no pole of $\Gamma(b_j+\beta_j s)$ for $j=1,2,\ldots,m$ coincides with any pole of of $\Gamma(1-a_j+\alpha_j s)$ for $j=1,2,\ldots,n$.
- C is a contour in the complex s-plane from $\omega-i\infty$ to $\omega+i\infty$ such that $(b_j+k)/\beta_j$ and $(a_j-1-k)/\alpha_j$ lie to the right and left of C, respectively.

References

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- Fox, C. "The G and H-Functions as Symmetrical Fourier Kernels." Trans. Amer. Math. Soc. 98, 395-429, 1961.
- Prudnikov, A. P.; Marichev, O. I.; and Brychkov, Yu. A. The Fox H-Function §8.3 in Integrals and Series, Vol. 3: More Special Functions. Newark, NJ: Gordon and Breach, pp. 626-629, 1990.
- Mathai, A. M.; Saxena, Ram Kishore; Haubold, Hans J. (2010), The H-function, Berlin, New York: Springer-Verlag.

Special cases

The generalized Bessel-Maitland function

$$J^{\lambda}_{\mu,\nu}(z) = H^{1,3}_{1,1}\left[\frac{z^2}{4}\middle|\begin{array}{l}\left(\lambda+\frac{\nu}{2},1\right)\\ \left(\lambda+\frac{\nu}{2},1\right),\left(\frac{\nu}{2},1\right),\left(\mu\left(\lambda+\frac{\nu}{2}-\lambda-\nu,\mu\right)\right)\end{array}\right]$$

The Wright generalized hypergeometric functions

$${}_p\Psi_q\left[z\middle|\begin{array}{c}(a_p,A_p)\\(b_q,B_q)\end{array}\right] = H^{p,q+1}_{1,p}\left[-z\middle|\begin{array}{l}(1-a_1,A_1)\ldots(1-a_p,A_p)\\(0,1),(1-b_1,b_1),\ldots(1-b_q,b_q)\end{array}\right]$$

More special cases

The Meijer G-function

$$G_{p,q}^{m,n}\left[z\left|\begin{array}{c}(a_1,\ldots,a_p)\\(b_1,\ldots,b_q)\end{array}\right.\right]=H_{m,n}^{p,q}\left[-z\left|\begin{array}{c}(a_1,1)\ldots(a_p,1)\\(b_1,1),\ldots(b_q,1)\end{array}\right.\right]$$

The Generalized Mittag-Leffler function

$$E_{\alpha,\beta}^{\gamma}(-z)=\frac{1}{\Gamma(\gamma)}H_{1,2}^{1,1}\left[z\left|\begin{array}{c}(1-\gamma,1)\\(0,1),(1-\beta,\alpha)\end{array}\right.\right]$$

Even more special cases

The MacRobert's E-function

$$E\left(\alpha_1,\ldots,\alpha_p;\beta_1,\ldots\beta_q;z\right)=H^{p,1}_{q+1,p}\left[z\left|\begin{array}{l}(1,1),(\beta_1,1),\ldots,(\beta_q,1)\\(\alpha_1,1),\ldots,(\alpha_p,1)\end{array}\right.\right]$$

The Whittaker function

$$W_{k,m}(z)=z^{-\rho}e^{z/2}H^{1,2}_{2,0}\left[\frac{z^2}{4}\left|\begin{array}{l}(\rho-k+1,1)\\\left(\rho+m+\frac{1}{2}\right),\left(\rho-m+\frac{1}{2},1\right)\end{array}\right.\right]$$

Structured photonic band gap reservoirs

- Discontinuity in the distribution of frequency modes
- New phenomena in atom-cavity interactions (oscillatory relaxation)

Special reservoir with structured photonic band gap

Continuous spectral density

$$J_\alpha(\omega) = \frac{2A(\omega-\omega_0)^\alpha \Theta(\omega-\omega_0)}{a^2+(\omega-\omega_0)^2}, \quad A>0, \quad a>0, \quad 1>\alpha>0$$

- PBG edge in the qubit transition frequency
- sub-ohmic at low frequencies $\omega \sim \omega_0$
- inverse power law for $\omega \gg \omega_0$ (similar to Lorentz)

$$\begin{aligned} J_\alpha(\omega) &\approx 2A/a^2(\omega-\omega_0)^\alpha \ \text{ for } \ \omega \to \omega_0^+ \\ J_\alpha(\omega) &\approx 2A\omega^{\alpha-2}, \ \text{ for } \ \omega \to +\infty \end{aligned} \tag{1}$$

Lorentzian type and PBG spectral densities

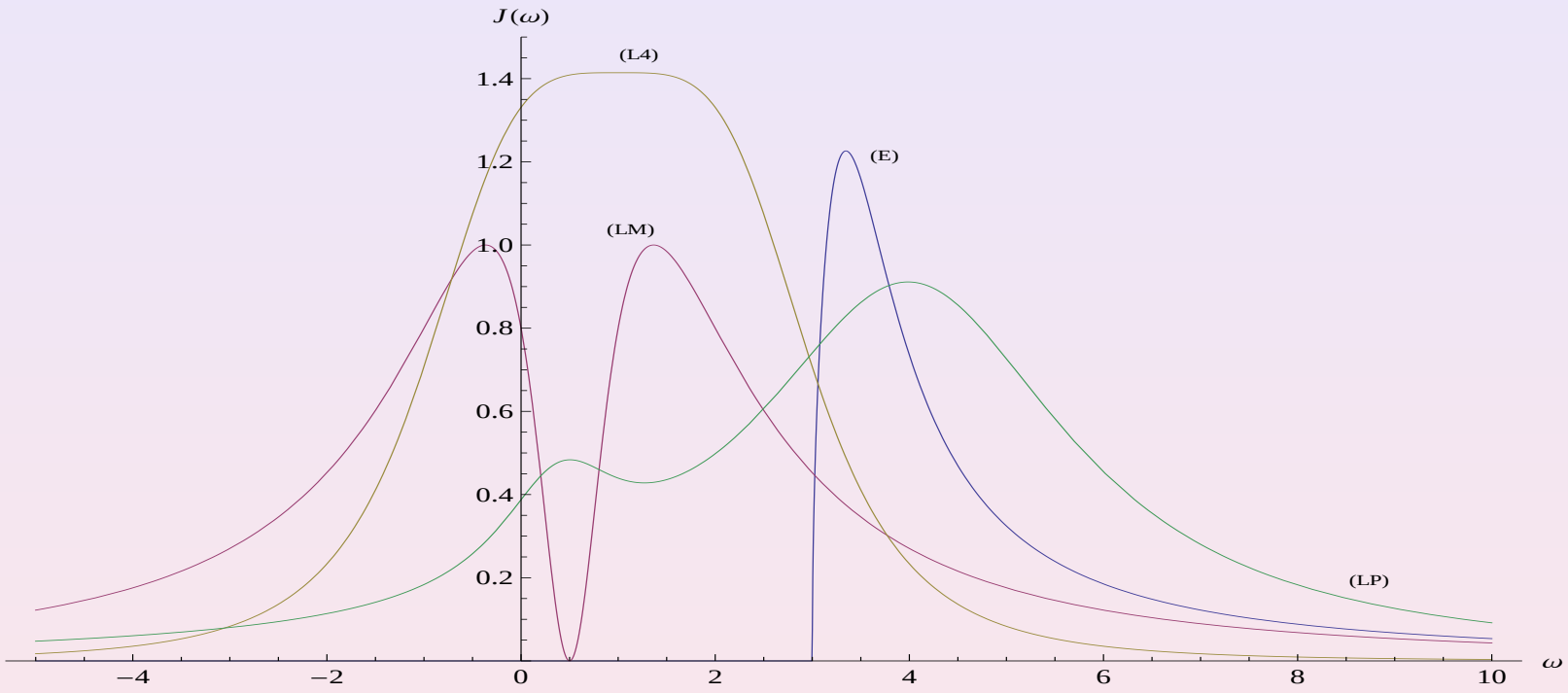


Figure: Various forms of spectral densities. The curve (LP) represents $\tilde{J}_{L_+}(\omega)$, the sum of two Lorentzians; (LM) is $\tilde{J}_{L_-}(\omega)$, the difference of two Lorentzians with PBG in the resonance frequency; $(L4)$ represents $\tilde{J}_{L'}(\omega)$ while (E) represents $J_E(\omega)$ with a PBG.

Exact dynamics of the qubit

Exact density matrix evolution:

$$\rho_{1,1}(t) = \rho_{1,1}(0)\,|G_\alpha(t)|^2\,, \quad \rho_{1,0}(t) = \rho^*_{0,1}(t) = \rho_{1,0}(0)\,e^{-\imath\omega_0 t}\,G_\alpha(t)$$

Exact result

$$\begin{aligned} G_\alpha(t) \;=\; & \sum_{n=0}^{\infty}\sum_{k=0}^{n} \frac{(-1)^n\, z_\alpha^k\, z_0^{n-k}\, t^{3n-\alpha k}}{k!(n-k)!} \\ & \times\left(H^{1,1}_{1,2}\left[z_1 t^2 \middle| \begin{matrix} (-n,1) \\ (0,1)\,,(\alpha k-3n,2) \end{matrix} \right] \right. \\ & \left. - a^2 t^2 H^{1,1}_{1,2}\left[z_1 t^2 \middle| \begin{matrix} (-n,1) \\ (0,1)\,,(\alpha k-3n-2,2) \end{matrix} \right] \right) \end{aligned}$$

The special case $\alpha = 1/2$ and the Eulerian dynamics (1)

$$J_E(\omega) = \frac{2A(\omega - \omega_0)^{1/2}\,\Theta(\omega - \omega_0)}{a^2 + (\omega - \omega_0)^2}$$

Exact dynamics (linear combination of Euler Incomplete Gamma functions)

$$\begin{aligned}
\rho_{1,1}(t) &= 1 - \rho_{0,0}(t) = \rho_{1,1}(0)\,|G_E(t)|^2 \\
\rho_{1,0}(t) &= \rho_{0,1}^*(t) = \rho_{1,0}(0)\,e^{-\imath\omega_0 t}\,G_E(t)
\end{aligned}$$

The special case $\alpha = 1/2$ and the Eulerian dynamics (2)

$$G_E(t) = \frac{1}{\sqrt{\pi}} \sum_{l=1}^{4} R\left(z_l\right) z_l \, e^{z_l^2 t} \, \Gamma\left(1/2, z_l^2 t\right)$$

where

- $$R(z) = \frac{(1-\imath)\left(a^{1/2}+z\right)\left(\imath a^{1/2}+z\right)}{2z\left((1+\imath)\,a + 3a^{1/2}z + 2\,(1-\imath)\,z^2\right)}$$
- z_1, z_2, z_3, z_4 roots of

$$Q\left(z_l\right) = \pi\sqrt{2/a}A + \imath a z_l^2 + (1+\imath)\,a^{1/2} z_l^3 + z_l^4 = 0, \quad l = 1,2,3,4$$

Giraldi F. and Petruccione F. (2010) *arXiv:1011.0059*

Longtime behaviour

Asymptotic expansion identifies

- time scale τ
- Decoherence factor D

such that for time scales $t \ll \tau$

$$G(t) \approx Dt^{-3/2}, \quad \text{for} \quad t \to \infty$$

Asymptotic form of ρ ($t \to \infty$)

$$\begin{aligned} \rho_{1,1}(t) &\approx \rho_{1,1}(0)|D|^2 t^{-3} \\ \rho_{1,0}(t) &\approx \rho_{1,0}(0)\exp(-i\omega_0 t)t^{-3/2} \end{aligned}$$

Lorentzian vs Eulerian relaxation

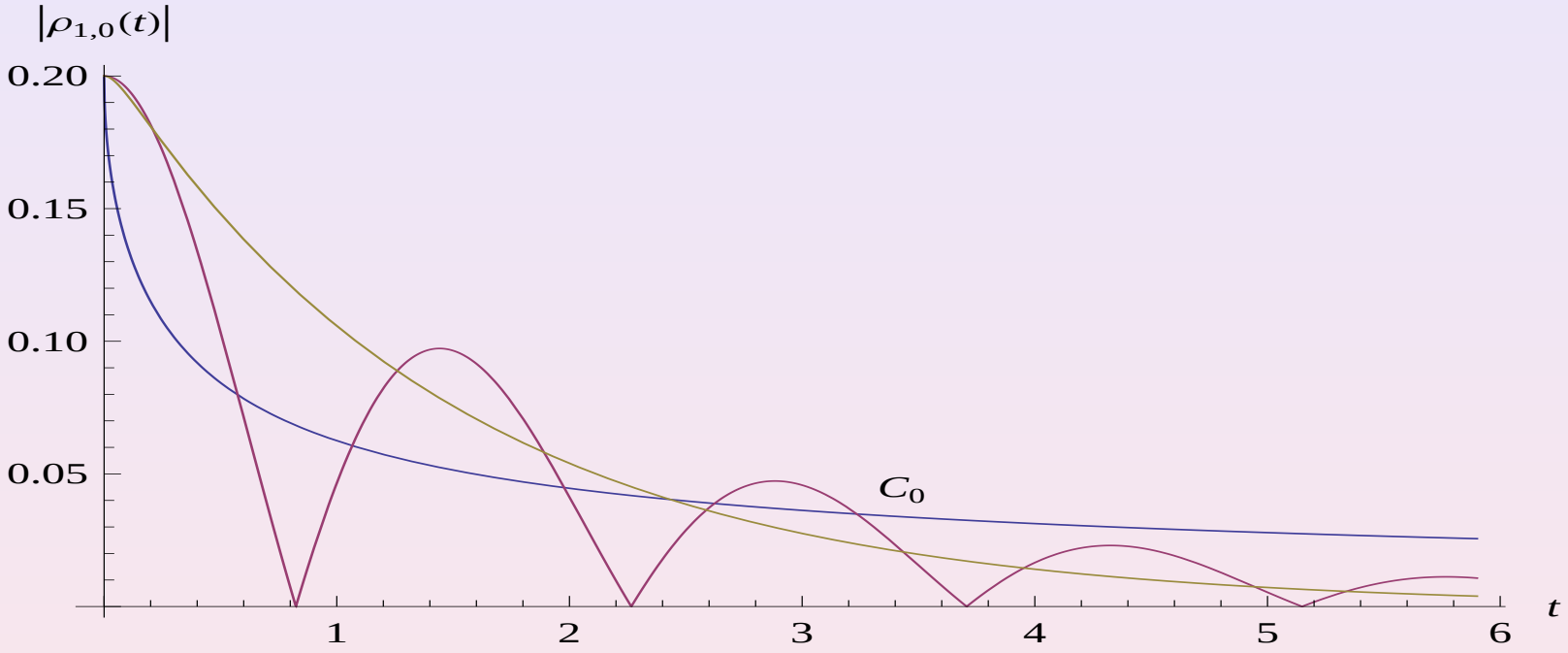


Figure: The time evolution of coherent term, $|\rho_{1,0}(t)|$, for a reservoir, described by either $\tilde{J}_L(\omega)$, both in strong coupling regime (red line) and weak coupling regime (yellow line), or $J_E(\omega)$ (blue line) spectral density function, respectively.

Exponential vs inverse power law

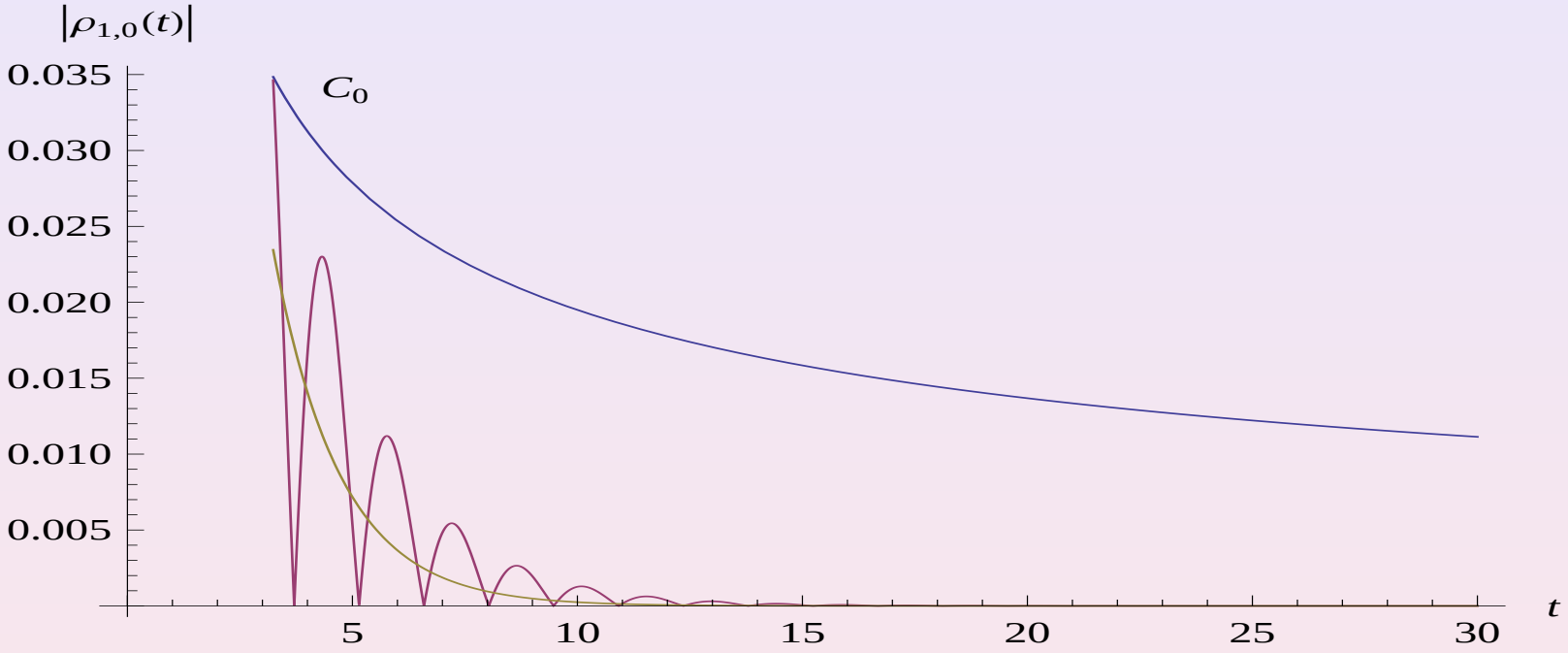


Figure: The relaxation of coherent term, $|\rho_{1,0}(t)|$, over long time scales, $t \gg 1$, $\tau \simeq 0.974$, $\tau_B = 1$ in strong coupling regime, $\tau_B = 0.05$ in weak coupling regime, of the reduced density matrix of a qubit, interacting with a reservoir, described by either $\tilde{J}_L(\omega)$, both in strong coupling regime (red line) and weak coupling regime (yellow line), or $J_E(\omega)$ (blue line) spectral density function.

The general case

$$J_\alpha(\omega) = \frac{2A(\omega-\omega_0)^\alpha \Theta(\omega-\omega_0)}{a^2+(\omega-\omega_0)^2}, \quad A>0, \quad a>0, \quad 1>\alpha>0$$

Time scale for inverse power law behaviour:

$$\tau_\alpha = \max\left\{1, \left|\frac{3}{z_0}\right|^{1/3}, \left|3\frac{z_\alpha}{z_0}\right|^{1/\alpha}, 3\left|\frac{z_1}{z_0}\right|\right\}$$

where

- $z_0 = \imath\pi A a^\alpha \cos(\pi\alpha/2)$
- $z_\alpha = -2\imath\pi A e^{-\imath\pi\alpha/2}\csc(\pi\alpha)$
- $z_1 = \pi A a^{\alpha-1}\sec(\pi\alpha/2) - a^2$

Towards $1/t$ qubit decoherence

Time scales $t \gg \tau_\alpha$:

$$G_\alpha(t) \sim -\mathcal{D}_\alpha \, t^{-1-\alpha}, \qquad t \to +\infty, \qquad 1 > \alpha > 0$$

where

$$\mathcal{D}_\alpha = \frac{2\,\imath\,\alpha\, a^{2(1-\alpha)} e^{-\imath\pi\alpha/2} \csc(\pi\alpha) \sec^2(\pi\alpha/2)}{\pi A\, \Gamma(1-\alpha)}$$

Exact dynamics of the qubit over long time scales

$$\begin{aligned} \rho_{1,1}(t) &= 1 - \rho_{0,0}(t) \sim \rho_{1,1}(0)\, |\mathcal{D}_\alpha|^2\, t^{-2-2\alpha} \\ \rho_{1,0}(t) &= \rho^*_{0,1}(t) \sim \rho_{1,0}(0)\, \mathcal{D}_\alpha\, e^{-\imath\omega_0 t}\, t^{-1-\alpha} \end{aligned}$$

Giraldi F. and Petruccione F. (2010) *arXiv:1011.0938*

Spontaneous emission of an excited atom

Total Hamiltonian: $H = H_A + H_E + H_I$

$$H_A = \omega_0 |1\rangle_{a\,a}\langle 1|, \qquad H_E = \sum_{k=1}^{\infty} \omega_k \, b_k^\dagger b_k,$$

$$H_I = \imath \sum_{k=1}^{\infty} g_k \left(b_k^\dagger \otimes |\,0\rangle_{a\,a}\langle 1| - b_k \otimes |1\rangle_{a\,a}\langle 0| \right).$$

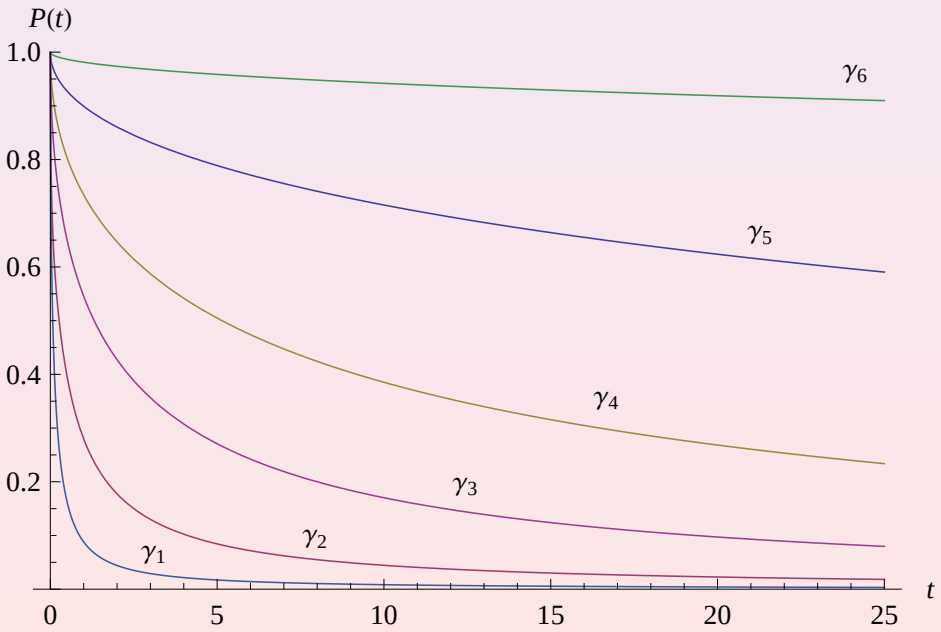
Initial state of the system

$$|\Psi(0)\rangle = |1\rangle_a \otimes |0\rangle_E$$

Time evolution of the population

The case $\alpha = 1/2$

$$P(t) = \frac{1}{\pi} \left| \sum_{l=1}^{4} \chi_l \, R(\chi_l) \, e^{\chi_l^2 t} \, \Gamma\left(1/2, \chi_l^2 t\right) \right|^2 .$$



γ_6: $A = 1$, $a = 1000$

γ_5: $A = 1$, $a = 100$

γ_4: $A = 5$, $a = 70$

γ_3: $A = 7$, $a = 35$

γ_2: $A = 5$, $a = 10$

γ_1: $A = 1$, $a = 1$,

Time evolution of the population (2)

The general case

$$P_\alpha(t) = \left| \sum_{n=0}^{\infty} \sum_{k=0}^{n} \frac{(-1)^n z_\alpha^k z_0^{n-k} t^{3n-\alpha k}}{k!(n-k)!} \left(H_{1,2}^{1,1}\left[z_1 t^2 \middle| \begin{matrix} (-n,1) \\ (0,1),(\alpha k - 3n, 2) \end{matrix} \right] - a^2 t^2 H_{1,2}^{1,1}\left[z_1 t^2 \middle| \begin{matrix} (-n,1) \\ (0,1),(\alpha k - 3n - 2, 2) \end{matrix} \right] \right) \right|^2$$

For $t \gg \tau_\alpha$

$$P_\alpha(t) \sim \zeta_\alpha \, t^{-2(1+\alpha)}, \quad t \to +\infty, \qquad 1 > \alpha > 0,$$

where

$$\zeta_\alpha = \frac{4\,\alpha^2\, a^{4(1-\alpha)} \csc^2(\pi\alpha) \sec^4(\pi\alpha/2)}{\pi^2 A^2 \left(\Gamma(1-\alpha)\right)^2}.$$

Spontaneous emission of an excited TLA in the presence of N-1 TLAs in the ground state

The Dicke model

$$H_N = \sum_{k=1}^{\infty} (\omega_k - \omega_0)\, b_k^\dagger b_k + \imath \sum_{k=1}^{\infty} g_k \left(J_{1,0}\, b_k^\dagger - J_{0,1}\, b_k \right)$$

where

- $J_{l,m} = \sum_{n=1}^{N} |\, l\rangle_{(n)\,(n)}\langle m|, \qquad l, m = 0, 1,$
- $J^2 = J_3^2 + (J_{2,1} J_{1,2} + J_{1,2}\, J_{2,1})\,/2$
- $J_3 = (J_{2,2} - J_{1,1})\,/2 \qquad J_3|J, M\rangle = M|J, M\rangle$

The superradiant states (initial condition): $|J, M = 1 - J\,\rangle$

Ref: S. John and T. Quang, Phys. Rev. A 50 (1994) 1764.

The exact decay

$$P_{N,\alpha}(t) = \left| \sum_{n=0}^{\infty} \sum_{k=0}^{n} \frac{(-N)^n \, z_\alpha^k \, z_0^{n-k} \, t^{3n-\alpha k}}{k!(n-k)!} \times \left(H_{1,2}^{1,1} \left[z_{N,1} \, t^2 \middle| \begin{matrix} (-n,1) \\ (0,1), (\alpha k - 3n, 2) \end{matrix} \right] - a^2 t^2 H_{1,2}^{1,1} \left[z_{N,1} \, t^2 \middle| \begin{matrix} (-n,1) \\ (0,1), (\alpha k - 3n - 2, 2) \end{matrix} \right] \right) \right|^2$$

$$z_{N,1} = \pi \, A \, N \, a^{\alpha-1} \sec(\pi\alpha/2) - a^2, \qquad z_{N,0} = N \, z_0, \qquad z_{N,\alpha} = N \, z_\alpha$$

Time scales and critical number of atoms for inverse power laws

The long time scale:

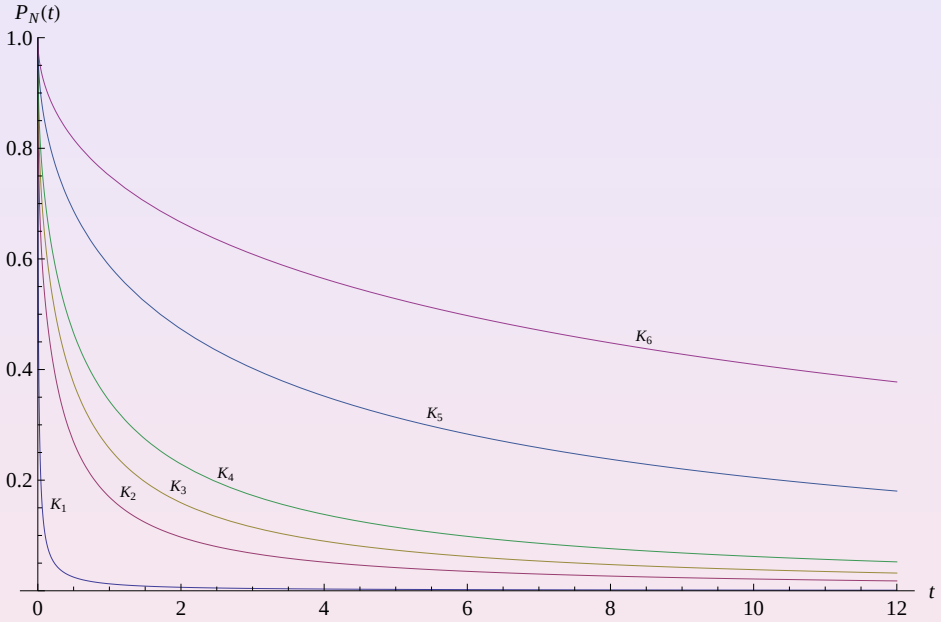
$$t \gg \tau_{N,\alpha}, \quad \tau_{N,\alpha} = \max\left\{1, \left|\frac{3}{z_{N,0}}\right|^{1/3}, \left|3\,\frac{z_\alpha}{z_0}\right|^{1/\alpha}, 3\left|\frac{z_{N,1}}{z_{N,0}}\right|\right\}$$

$$P_{N,\alpha}(t) \sim \zeta_{N,\alpha}\, t^{-2(1+\alpha)}, \quad 1 > \alpha > 0$$

$$\zeta_{N,\alpha} = \frac{4\,\alpha^2\, a^{4(1-\alpha)} \csc^2(\pi\alpha) \sec^4(\pi\alpha/2)}{\pi^2 A^2 N^2 \left(\Gamma(1-\alpha)\right)^2}$$

$$N \gg N_\alpha^{(\star)} \Rightarrow \zeta_{N,\alpha} \ll 1, \quad N_\alpha^{(\star)} = \left[\frac{2\,\alpha\, a^{2(1-\alpha)} \csc(\pi\alpha) \sec^2(\pi\alpha/2)}{\pi A\, \Gamma(1-\alpha)}\right]$$

Ref: F. Giraldi and F. Petruccione (2010) *ArXiv:1011.3014*



Parameters: $a = 20,\ A = 1/3$

K6: N=2

K5: N=7

K4: N=30

K3: N=50

K2: N=90

K1: N=1000

Suppression of trapping for large N

Critical number: $N^*_{1/2} = 21$

Thank you for your attention!

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