Van Hove Limit for Infinitely Extended Open Quantum Systems

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29th November 2010

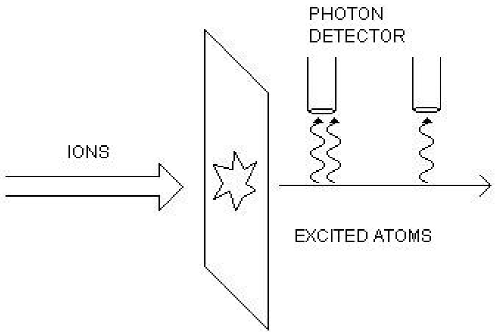
Outline

- The Markovian Approach and Davies generators
- Our Generator : Quantum Fokker-Planck Equation
- Example : a Quantum Particle in 3D Space
- Proposal : a Quantum Collisionless Boltzmann Equation
- Outlook on Quantum Brownian Motion
- Conclusions

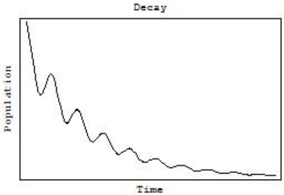
Collaborations

- Prof. F. Rossi (Dip. Fisica, Politecnico di Torino)
- Prof. H. Fujita Yashima (Dip. Mat., Universitá di Torino)
- Prof. C-A Pillet (CPT - UMR 6207 et Université de Toulon)
- Prof. V. Gritsev (Dép. Phys., Université de Fribourg)

An experiment to start with!



Need of a Quantum Theory of Relaxation Phenomena



- Coherent superposition of system eigenstates

$\Rightarrow$ non-trivial interplay between Coherent Dynamics and Energy-Relaxation/Decoherence

Contact with microscopic quantum description at large times

- The fundamental equations governing the basic laws of Physics are time reversible and not dissipative.
- Macroscopic irreversible equations obtained through
  - averaging over microscopic degrees of freedom (stochasticity)
  - energy-time scale separation (μeV versus meV, etc.)
  - neglecting recollisions (Markovicity)

Van Hove Limit in Quantum Open Systems

- $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_B \qquad H_0 = H_S \otimes 1 + 1 \otimes H_B$
- $H' = Q \otimes \Phi, \qquad H_\lambda = H_0 + \lambda H'$

System observables in Heisenberg picture

- The state on $\mathcal{H}$ is $\rho = \rho_s \otimes \sigma_\beta$
- At time t, $O^\lambda(t) = e^{iH_\lambda t}\, O_S \otimes 1\, e^{-iH_\lambda t}$
- We measure $\langle O^\lambda(t) \rangle_\rho = \mathrm{tr}[\rho\, O^\lambda(t)] = \mathrm{tr}[\rho_S\, O_S^\lambda(t)]$ where
- $O_S^\lambda(t) = P_0\ O^\lambda(t)$ system observable at time t
- $P_0\ X \otimes Y = tr(\sigma_\beta Y)\ X \otimes \mathit{1}$ partial trace projection

Markovian Approximation in the Van Hove Limit

- Define $W_t^\lambda O_S := O_S^\lambda(t)$ system evolution superoperator
- Expect $W_t^\lambda \sim \exp\{\mathbb{L}_\lambda t\}, \quad 0 \le t \le \lambda^{-2}\overline{\tau}, \quad \lambda \sim 0$

[1] L. Van Hove, Physica **21** 517 (1955)

Exact System Evolution : the Memory Kernel

Formulation on operator spaces

- $\mathcal{B} = \mathcal{B}_0 \oplus \mathcal{B}_1$ Banach spaces $\mathcal{B}_i = P_i(\mathcal{B})$, $\quad P_1 = 1 - P_0$
- $Z\,O = i[H_0, O]$ and $A\,O = i[H', O]$ **Liouvillians**
- $W_t^\lambda = P_0 \exp\{(Z + \lambda A)t\}|_{\mathcal{B}_0}$ **subsystem evolution**

The Nakajima-Zwanzig master equation

$$W_t^\lambda = X_t^\lambda + \lambda^2 \int_0^t dt_1 \int_0^{t_1} dt_2\, X_{t-t_1}^\lambda A_{01} U_{t_1-t_2}^\lambda A_{10} W_{t_2}^\lambda$$

- $A_{ij} = P_i A P_j$ splitted interaction
- $U_t^\lambda = \exp\{(Z + \lambda A_{00} + \lambda A_{11})t\}, \qquad X_t^\lambda = P_0 U_t^\lambda$

[1] Nakajima, S., Prog. Theor. Phys. **20(6)** 948-959 (1958)

[2] Zwanzig, R, J. Chem. Phys. **33** 1338 (1960)

The Born-Markov approximation: Davies generator

Markovian Hypotheses (Bounds on Dyson Expansion)

$$\int_0^{\lambda^{-2}\overline{\tau}} dx \, \|A_{01} U_x^\lambda A_{10}\| < C, \quad |\lambda| < 1$$

$$\forall \overline{\tau} > 0, \; \lim_{\lambda \to 0} \int_0^{\lambda^{-2}\overline{\tau}} dx \, \|A_{01}(U_x^\lambda - U_x) A_{10}\| = 0$$

Davies Markovian Approximation Theorem (MAT)

- $\forall \overline{\tau} > 0 \quad \lim_{\lambda \to 0} \sup_{0 \le t \le \lambda^{-2}\overline{\tau}} \|W_t^\lambda - \exp\{\mathbb{L}_\lambda t\}\| = 0$
- with $\mathbb{L}_\lambda := Z_0 + \lambda A_{00} + \lambda^2 K_D$, and
- $K_D = \int_0^\infty dr \, U_{-r} A_{01} U_r A_{10}$ Davies generator

- K_D well defined for arbitrary H_S spectra

[1] E. B. Davies, *Markovian Master Equations II*, Math. Ann. **219** 147-158 (1976)

Confined Systems : Davies averaged generator[1]

Time averaging map $\natural$

$$K^{\natural} = \lim_{T \to +\infty} \frac{1}{2T} \int_{-T}^{T} dq \, U_q K U_{-q}$$

Then $\mathbb{L}_\lambda = Z_0 + \lambda^2 K_D^{\natural}$

- satisfies MAT iff K_D does
- incorporates Pauli Master Equation as $[Z_0, K_D^{\natural}] = 0$
- describes resonances of the Liouvillian (Fermi Golden Rule)[2,3]
- generates a Quantum Dynamical Semigroup

but

- only if P_0 is a partial trace
- only when $A_{00} = 0$ (no average forces on the system)
- $K^{\natural}$ well defined only if Z_0 has discrete spectrum

[1] E. B. Davies, Commun. Math. Phys. **39** 91-110 (1974)
[2] Jaksic V., Pillet C.-A., Ann. Inst. H. Poincare Phys. Theor. **67** 425-445 (1997)
[3] Derezinski J., Jaksic V., J. Stat. Phys. **116** 411-423 (2004)

Infinitely Extended systems : K_D is the only candidate

all of $\mathbf{R}^3$. The Markovian limit for systems with an H_S with a discrete spectrum, is, in essence, understood, whereas the case of an H_S with a continuous spectrum still presents certain difficulties.

[1] H. Sphon, Rev. Mod. Phys. **53** 3 (1980)

K_D employed only under severe restrictions

ABSTRACT. — We consider a non-relativistic quantum mechanical particle in an external potential well, coupled to an infinite free quantum field. We prove rigorously that with certain cut-offs and in the weak coupling limit, the particle decays exponentially between its bound states as predicted by perturbation theory. We also prove the existence of a « dyna-

[2] E.B. Davies, Ann. Inst. Henri Poincaré **28** 1 (1978)

Why K_D is so bad?

- It does not generate a proper QDS

  [4] Dümcke R and Spohn H, Z. Phys. B **34** 419 (1979)

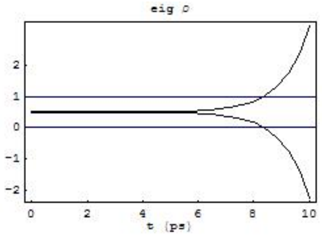
- is it really a big problem after all, or just some transient?

Failure of K_D approximation at large times

Case of a two-level quantum-dot system in a thermal bosonic environment

One particle sector

$$\rho = \begin{pmatrix} \langle a|\rho|a\rangle & \langle a|\rho|b\rangle \\ \langle b|\rho|a\rangle & \langle b|\rho|b\rangle \end{pmatrix} = \begin{pmatrix} f_a & p \\ p^* & f_b \end{pmatrix}$$



- Very small perturbation of thermal distribution at $t = 0$
- Characteristic interlevel splitting: 30 meV
- Very high temperatures!

- Analytically solved: divergences don't come from numerics!
- **Totally unphysical results for large times/steady states**

[1] Taj D., Iotti R.C., Rossi F. , Eur. Phys. J. B **72** 3 (2009)

The main idea : symmetry could recover probabilities

I remember once when I was in Copenhagen, that Bohr asked me what I was working on and I told him I was trying to get a satisfactory relativistic theory of the electron, and Bohr said 'But Klein and Gordon have already done that!' That answer first rather disturbed me. Bohr seemed quite satisfied by Klein's solution, but I was not because of the negative probabilities that it led to. I just kept on with it, worrying about getting a theory which would have only positive probabilities.

Conversation between Dirac and J. Mehra, March 28, 1969, quoted by Mehra in *Aspects of Quantum Theory*, ed. by A. Salam and E. P. Wigner (Cambridge University Press, Cambridge, 1972).

Figure: S. Weinberg "The Quantum Theory of fields", vol 1, Cambridge University Press (1995)

Probabilities must be positive!

- It could help in getting a good (unique?) evolution equation
- **Hidden time symmetries in the memory kernel could imply positive probabilities!**

The Van Hove Limit: a new approach

The Nakajima-Zwanzig master equation

$$W_t^\lambda = X_t^\lambda + \lambda^2 \int_0^t dt_1 \int_0^{t_1} dt_2 \, X_{t-t_1}^\lambda A_{01} U_{t_1-t_2}^\lambda A_{10} W_{t_2}^\lambda$$

Davies' change of variable in the integral kernel

$$\begin{pmatrix} \sigma \\ r \end{pmatrix} = \begin{pmatrix} 0 & \lambda^2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \end{pmatrix}$$

- linear homogeneous
- λ^2 jacobian

Our change of variable in the integral kernel

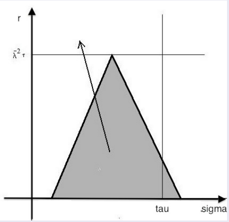
$$\begin{pmatrix} \sigma \\ r \end{pmatrix} = \begin{pmatrix} \lambda^2/2 & \lambda^2/2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} + \begin{pmatrix} \lambda^2 q \\ 0 \end{pmatrix} \quad \text{for some } q \in \mathbb{R}$$

(we will remove the q-asymmetry in a second step)

Dynamical Scattering Time T_λ

Time rescaled interaction picture: $W_\tau^{\lambda,i} = X^\lambda_{-\lambda^{-2}\tau} W^\lambda_{\lambda^{-2}\tau}$

$$W_\tau^{\lambda,i} = 1 + \iint_{\mathcal{D}(\lambda,q)} d\sigma\, dr\, X^\lambda_{-\lambda^{-2}\sigma - \frac{r}{2} - q} A_{01} U_r^\lambda A_{10} X^\lambda_{\lambda^{-2}\sigma - \frac{r}{2} + q} W^{\lambda,i}_{\sigma + \lambda^2(\frac{r}{2} + q)}$$



- Let $T_\lambda \approx |\lambda|^{-\xi}, \quad \lambda \sim 0, \quad 0 < \xi < 2$
- e.g. $T_\lambda = \left(|\lambda| \|P_0 A^2 P_0\|^{1/2}\right)^{-1}$
  Dynamical Scattering Time

Memory effects removal under Markovian Hypotheses

- $\xi > 0 \Rightarrow \quad \iint_{\mathcal{D}(\lambda,q)} d\sigma\, dr \approx \int_0^\tau d\sigma \int_0^\infty dr\, e^{-\left(\frac{r}{2}\right)^2 / T_\lambda^2}, \quad \lambda \sim 0$
- $\xi < 2 \Rightarrow \quad W^{\lambda,i}_{\sigma + \lambda^2(\frac{r}{2} + q)} \approx W_\sigma^{\lambda,i}, \quad \lambda \sim 0$

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Averaging with Dynamical Scattering Time

Averaging among our generators

$$K_{(q,T)} = \int_0^{\infty} dr\, e^{-\frac{(r/2)^2}{T^2}}\; U_{-\frac{r}{2}+q} A_{01} U_r A_{10} U_{\frac{r}{2}-q} \;= U_q\, K_{(0,T)}\, U_{-q}$$

- $\overline{\{K_{(q,T)}\}_{q\in\mathbb{R}}}$ corresponds to $K^{\natural}_{(0,T)}$: use gaussian with $\sigma = T_\lambda$!

Our Dynamical Time averaged generator

$$K_T = P_0 \left\{ \int_{-\infty}^{+\infty} dt_1\, \Phi(t_1) \int_{-\infty}^{t_1} dt_2\, \Phi(t_1) \right\} P_0$$

$$\Phi(t) = \sqrt{\delta_T}(t)\, U_{-t}(A - A_{00}) U_t, \qquad \delta_T(t) = \frac{1}{\sqrt{2\pi}T} e^{-\frac{t^2}{2T^2}}$$

Results under the same Markovian Hypotheses of Davies

- MAT for $\mathbb{L}_\lambda = Z_0 + \lambda A_{00} + \lambda^2 K_{T_\lambda}$ $(0 \le t \le \lambda^{-2}\overline{\tau}, \quad \lambda \sim 0)$
- $\mathbb{L}_\lambda$ always well defined $\forall \lambda \neq 0$, independently of Z_0 spectrum!
- If $\|P_0\| = 1$ then $\exp\{\mathbb{L}_\lambda t\}$ **is a contraction**!!!
- $\lim_{\overline{\tau}\to+\infty} K_{|\lambda|^{-1}\overline{\tau}} = K^{\natural}_D$ when $\exists$, recovering Davies

K_T generates a QDS on Operator Algebras

Let $\mathcal{B}, \mathcal{B}_0$ be Operator Algebras with identity

- Let $P_0 : \mathcal{B} \to \mathcal{B}_0$ Conditional Expectation
- Let $P_0(i[H_\lambda, \cdot])$ generate automorphisms ($H_\lambda = H_0 + \lambda H'$)

"The" Quantum Fokker-Planck Equation

$$\partial_t X = P_0(i[H_\lambda, X]) + \lambda^2 i \left[\int \frac{d\omega}{2\pi\omega} P_0(\widetilde{\mathcal{L}}^\dagger_{\lambda\omega} \widetilde{\mathcal{L}}_{\lambda\omega}), X \right] - \frac{\lambda^2}{2} \{ P_0(\widetilde{\mathcal{L}}_\lambda \widetilde{\mathcal{L}}_\lambda), X \} + \lambda^2 P_0(\widetilde{\mathcal{L}}_\lambda X \widetilde{\mathcal{L}}_\lambda)$$

Dynamically Averaged Coupling

$$\mathcal{L}_{\lambda\omega} = \int_{-\infty}^{+\infty} dt \, \sqrt{\delta_{T_\lambda}}(t) \, e^{i\omega t} \, U_t(H') \qquad \widetilde{\mathcal{L}}_{\lambda\omega} := P_1(\mathcal{L}_{\lambda\omega})$$

A Free Quantum Particle in 3D Euclidean Space

Inelastically Coupled to a Fermionic Heath Bath

Limit Dynamics for $H_0 = H_S \otimes 1 + 1 \otimes H_B, \quad H' = Q \otimes \Phi$

- $h(t) = \mathrm{tr}[\sigma_\beta \Phi U_t(\Phi)] - \mathrm{tr}[\sigma_\beta \Phi]^2$, first order corrected!
- $A_{\omega,\lambda} = \int \frac{dt}{\sqrt{2\pi}} \sqrt{\delta_{T_\lambda}}(t)\, e^{i\omega t}\, e^{iH_S t}\, Q\, e^{-iH_S t}$

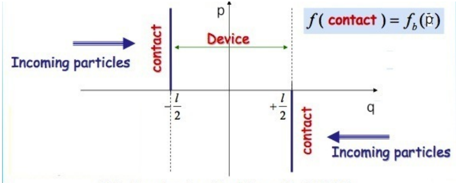
$$K_{T(\lambda)}X = -2\pi i \int \frac{d\omega}{\sqrt{2\pi}} s(\omega)\, [A^\dagger_{\omega,\lambda} A_{\omega,\lambda}, X] + 2\pi \int \frac{d\omega}{\sqrt{2\pi}} \hat{h}(\omega) \left(-\frac{1}{2} \left\{ A^\dagger_{\omega,\lambda} A_{\omega,\lambda}, X \right\} + A^\dagger_{\omega,\lambda} X A_{\omega,\lambda} \right)$$

- Markovian Hypotheses verified if $\int dt\ h(t)(1 + |t|^\epsilon) < \infty$
- For $H_S = \varepsilon(P) = \frac{P^2}{2}$ in 3D, $\langle p|Q|p'\rangle = q(\varepsilon_p, \varepsilon_{p'})$, $q \in S(\mathbb{R}^2)$, there $\exists\, L$ s.t. $\|T(\lambda)K_{T(\lambda)} - L\| \to 0$, and $[Z_0, L] = 0$.
- Thermal distributions of observables affiliated to H_S are stationary under L if furthermore $\hat{h} \in S(\mathbb{R})$.
- I found a Pauli Equation with FGR conditionally on $\rho_S = \rho_\beta$ ...

Abstract. We study the van Hove limit for master equations on a Banach space, and propose a contraction semigroup as limit dynamics. The generator has a Lindblad form if specialized to C^*-algebras, is always well defined irrespectively of the subsystem spectrum, includes first-order contributions, and returns Davies averaged generator, when the latter is defined. The theory is applied to the case of a free particle in contact with a heat bath.

[1] Taj D., Ann. Henri Poincaré Online First (2010)

Time Independent Mesoscopic Quantum Transport
A device modelling approach



W.R. Frensley, Rev. Mod. Phys. 62, 3 (1990)

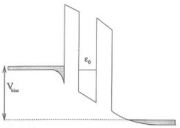

158 12. Transport in Mesoscopic Semiconductor Structures

Fig. 12.1. Double-barrier semiconductor heterostructure biased close to resonance, where charge carriers emerging from the source contact are matched to the energy of the quasibound state ε_0 in the quantum well. Occupied contact states are shown as hatched, and the band bending is due to charge accumulation or depletion

Some Existing Models in Perpendicular Quantum Transport

- Landauer-Buttiker[1] : NESS I-V
- Quantum Kinetics (Haug, Jaujo) : quantum truncation.
- Lattice models (Datta) : atomistic devices

[1] W. Aschbacher, V. Jaksic, Y. Pautrat, C.-A. Pillet, J. Math. Phys. **48** 032101-032129 (2007)

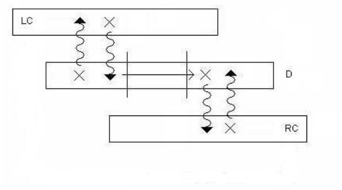
Want

- Full space and time resolution of charge density in device
- Boltzmann Transport Eq. with Up-wind BC's as $\hbar \to 0$

Quantum Transport: our model (1)

Physical ideas

- LC, RC, D all infinitely extended: **quantum non-locality**
- LC-D and RC-D **interactions spatially localized**



Physical Subsystem

- $\mathcal{F} = \mathcal{F}_l \otimes \mathcal{F}_d \otimes \mathcal{F}_r$
- $\langle O_l \otimes O_d \otimes O_r \rangle = \mathrm{Tr}[\overline{\sigma}_l O_l]\, \mathrm{Tr}[\overline{\sigma}_r O_r]\, O_d$

Spatial locality: interaction implemented

- $H'_z = \int C_z(x) \Psi^\dagger_z(x) \Psi_d(x)\, dx + h.c., \quad (z = l, r)$
- e.g. $C_l(x) = \theta(-(x + l/2)),\ C_l(x) = \theta(x - l/2)$

Quantum Transport: our model (2)

Limit dynamics: the many body equation

$$\partial_t X = i[H_d, X] + \lambda^2 \int dk\, g_z^{\pm}(k) \left(i \left[\int \frac{d\omega}{2\pi\omega} a^{\pm}(\Psi^z_{k\omega}) a^{\mp}(\Psi^z_{k\omega}), X \right] \right.$$
$$\left. -\frac{1}{2} \left\{ a^{\pm}(\Psi^z_k) a^{\mp}(\Psi^z_k),\, X \right\} + a^{\pm}(\Psi^z_k)\, X\, a^{\mp}(\Psi^z_k) \right)$$

- **Fermi-Dirac** $f^z_{\beta\mu}$, $\quad g_z^+ = f^z_{\beta\mu}$, $\quad g_z^- = 1 - f^z_{\beta\mu}$
- **Scattering probability amplitude** $\boldsymbol{\Psi}^{\mathbf{z}}_{\mathbf{k},\omega}(k') = \sqrt{\delta_{\varpi_\lambda}}(\omega^z_k - \omega^d_{k'} - \omega)\, \hat{C}_z(k, k')$

- **Excellent physical interpretation, and exactly solvable!**
- **Gaussian states are invariant** $\omega_G(a^\dagger(f_1) \cdots a^\dagger(f_n) a(g'_n) \cdots a(g_1)) = \delta_{n,n'} \det\langle f_i, G\, g_j \rangle$
- G obeys an associated **closed linear equation**

A Wigner Transport Equation (WTE) for a free device

The (impoper) Wigner Function on the "classical" phase space

- Classical picture : density $f(q,p)$
- Quantum picture : $f(q,p) = \int \frac{dr}{2\pi} e^{ipr} \langle q + \frac{r}{2} | G | q - \frac{r}{2} \rangle$

For a free device $h_d = \frac{P^2}{2m}$, the Eq. for G becomes

WTE $\quad \partial_t f = -p\,\partial_q f - \frac{1}{2}\{L^\lambda\,,\, f\}_{\star_\hbar} + S^\lambda$

- $L_l^\lambda(q,p) = L_l^\lambda(q) = \sqrt{2\pi}\lambda^2 T(\lambda)\,\theta[-(q + l/2)]$
- $S_l^\lambda(q,p) = L_l^\lambda(q) \int \frac{dp'}{\pi}\, f_{\beta\mu}^l(p')\, \frac{1}{\Delta p(q)}\, \mathrm{sinc}\left(\frac{p-p'}{\Delta p(q)}\right)$
- $\Delta p(q) = \frac{\hbar}{2} \frac{1}{|q + l/2|}$

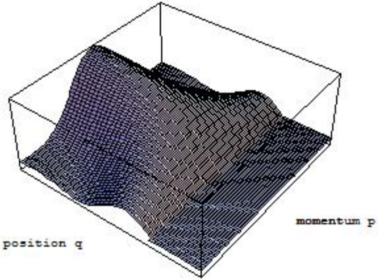
  $\Rightarrow$ classical scheme recovered for $\hbar \to 0$!

  $\Rightarrow$ classical scheme recovered for $q \to \pm\infty$!

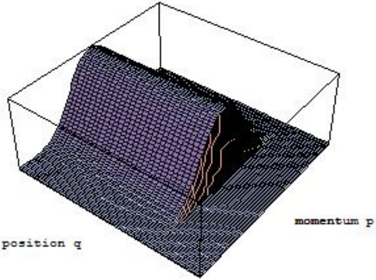
  $\Rightarrow$ Heisenberg principle appears very naturally!

A Wigner Transport Equation (WTE) for a free device

Quantum source



Classical Source



- $S_l^{\lambda}(q,p) = L_l^{\lambda}(q) \ \int \frac{dp'}{\pi} \ f_{\beta\mu}^{l}(p') \ \frac{1}{\Delta p(q)} \, \mathrm{sinc}\left(\frac{p-p'}{\Delta p(q)}\right)$
- $\Delta p(q) = \frac{\hbar}{2} \frac{1}{|q+l/2|}$

  $\Rightarrow$ classical scheme recovered for $\hbar \to 0$!

  $\Rightarrow$ classical scheme recovered for $q \to \pm\infty$!

  $\Rightarrow$ Heisenberg principle appears very naturally!

Summary of this section

We have proposed a WTE for mesoscopic time independent quantum transport (Quantum Collisionless Boltzmann Equation)

$$\partial_t f = -p\,\partial_q f - \frac{1}{2}\{L^\lambda\,,\,f\}_{\star_\hbar} + S^\lambda$$

Although still preliminary, our WTE

- is physically robust (guarantees positivity at all times)
- it's classical limit is a well known PDE (Collisionless Boltzmann Equation) with appropriate (Up-wind) BC's
- it's a limit dynamics of the exact hamiltonian evolution
- fully embodies quantum uncertainty principles
- the solution offers a full space and time resolution of device
- phonons and nonzero average forces could be accounted for...

Quantum Brownian Motion

Einstein's kinetic theory of the Brownian motion, based upon light water molecules continuously bombarding a heavy pollen, provided an explanation of diffusion from the Newtonian mechanics. Since the discovery of quantum mechanics it has been a challenge to verify the emergence of diffusion from the Schrödinger equation.

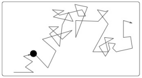


Figure 1: Brown's picture under the microscope

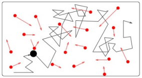


Figure 2: Einstein's explanation to Brown's picture

	CLASSICAL MECHANICS	QUANTUM MECHANICS
Stochastic dynamics (no memory)	Random walk (Wiener)	Random kick model with zero time corr. potential (Pillet, Schenker-Kang)
Hamiltonian particle in a random environment (one body)	Lorentz gas: particle in random scatterers (Kesten-Papanicolaou) (Komorowski-Ryzhik)	Anderson model or quantum Lorentz gas (Spohn, Erdős-Yau, Erdős-Salmhofer-Yau Disertori-Spencer-Zirnbauer)
Hamiltonian particle in a heat bath (randomness in the many-body data)	Einstein's kinetic model (Dürr-Goldstein-Lebowitz)	Electron in phonon or photon bath (Erdős, Erdős-Adami, Spohn-Lukkarinen De Roeck–Fröhlich)
Periodic Lorentz gas (randomness in the one-body initial data)	Sinai billiard (Bunimovich-Sinai)	Ballistic (Bloch waves, easy)
Many-body interacting Hamiltonian	Nonlinear Boltzmann eq (short time: Lanford)	Quantum NL Boltzmann (unsolved)

[1] L. Erdos, Lecture notes on Quantum Brownian Motion, Les Houches Summer School (2010)

The diffusion constant

Heuristically, for $N \gg 0$ interactions

- $(\delta x) \sim \sqrt{N}$
- $N \sim t$

$$(\delta x)^2 = D\ t, \qquad t \to \infty$$

Low relative momenta

$$\langle X^2 \rangle_\rho(t) = \mathrm{tr}(\rho(t) X^2) = \int dp\ \partial_r^2 [\rho_r](t)|_{r=0}(p)$$

- $[\rho_r](t)(p) := \left\langle p - \frac{r}{2} \right| \rho(t) \left| p + \frac{r}{2} \right\rangle$
- r: **relative momentum**

$\Rightarrow$ Only low relative momenta matter : need $[\rho_r]$ up to $o(r^2)$

A Translation Invariant Model

Model Hamiltonian for a Quantum Particle in $\mathbb{R}^d$

- $H_0 = \epsilon(P) \otimes 1 + 1 \otimes \int dq\, \omega_q\, b_q^\dagger b_q$
- $H' = \int dp_1 dp_2 dq\, |p_1\rangle\langle p_2| \otimes b_q\ \phi(q)\ \delta(p_1 - p_2 - q)\ + h.c.$

Relative Momenta Fibration of the Limit Dynamics

$$\partial_t [\rho]_r = L_r^\lambda\ [\rho]_r$$

- $\{f(P)\}$ is **left invariant** under $K_{T(\lambda)}$
- The zero fiber is the Pauli Equation

$$\partial_t f(p) = \lambda^2 \int dp'\ \{m(p,p')\, f(p') - m(p',p)\, f(p)\}$$

- with FGR transition rates

$$\begin{aligned} m(p,p') &= \{|\phi(p-p')|^2 N_{p-p'} \delta(\epsilon_p - \epsilon_{p'} - \omega_{p-p'}) \\ &+ |\phi(p'-p)|^2 (1 + N_{p-p'}) \delta(\epsilon_{p'} - \epsilon_p - \omega_{p'-p})\} \end{aligned}$$

Summary and Conclusions

Generalised Van Hove Limit

We studied weakly perturbed projected one-parameter group of isometries. We have found a generator that

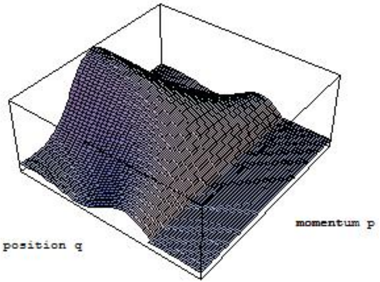
- satisfies MAT in the Van Hove Limit under Davies markovian hypotheses
- is always well defined, independently of subsystems details
- generates contractions and a QDS in operator algebras
- generalizes Davies generator

It furnishes a way to understand infinite open systems such as

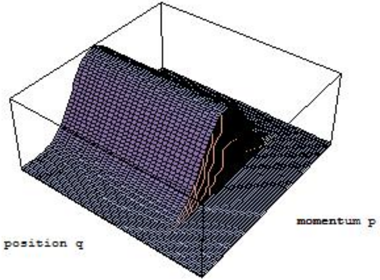
- A free particle in 3D locally coupled to a fermionic heath bath
- Nanodevices (Quantum Collisionless Boltzmann Equation)
- A free particle in 3D non-locally coupled to bosonic bath?

A Wigner Transport Equation (WTE) for a free device

Quantum source



Classical Source



- $S_l^\lambda(q,p) = L_l^\lambda(q) \int \frac{dp'}{\pi} f^l_{\beta\mu}(p') \frac{1}{\Delta p(q)} \operatorname{sinc}\left(\frac{p-p'}{\Delta p(q)}\right)$
- $\frac{1}{2}(L_l^\lambda \star_\hbar f + f \star_\hbar L_l^\lambda)(q,p) = L_l^\lambda(q) \int \frac{dp'}{\pi} f(q,p') \frac{1}{\Delta p(q)} \operatorname{sinc}\left(\frac{p-p'}{\Delta p(q)}\right)$

$\Rightarrow$ with only left reservoir, say, $f(q,p) = f^l_{\beta\mu}$ is stationary!