SPECTRAL THEORY AND MATHEMATICAL PHYSICS

Université de Cergy-Pontoise,

June 21 -24, 2016

Organisers : Laurent Bruneau, Thierry Daudé, Vladimir Georgescu

Webpage : *http://bruneau.u-cergy.fr/SpectralTheory-MathPhys-2016/main.html*

Program

This workshop will take place June 21-24 at the Université de Cergy-Pontoise , Campus St. Martin, Amphi Colloques, Bâtiment E - RdC.

Tuesday, June 21

14h00 – 14h55 : Jacob Schach Møller.
Local Spectral Deformation

15h05 – 16h00 : Matti Lassas.
Scattering in complex geometrical optics and the inverse problem for the conductivity equation

16h00 – 16h30 : Coffee, tea, cookies.

16h30 – 17h25 : Kais Ammari.
Schrödinger operator on some graphs : stabilization and dispersive effects

Wednesday, June 22

09h30 – 10h00 : Coffee, tea, croissants

10h00 – 10h55 : Clotilde Fermannian Kammerer.
Wigner measures and effective mass theorems.

11h05 – 12h00 : Spyros Alexakis.
Singularity formation in Black hole interiors.

12h00 – 14h00 : Lunch.

14h00 – 14h55 : Jan Derezinski.
Propagators on curved spacetimes

15h00 – 15h30 : Coffee, tea, cookies.

15h30 – 16h25 : Victor Nistor.
Cross-product algebras and essential spectra : applications to the N-body problem.

Thursday, June 23

09h30 – 10h00 : Coffee, tea, croissants

10h00 – 10h55 : Christian Gérard.
Aspects of Quantum Field Theory in blackhole spacetimes.

11h05 – 12h00 : Benoit Pausader.
Norm growth for the cubic nonlinear Schrodinger equation.

12h00 – 14h00 : Lunch.

14h00 – 14h55 : Jean-Marc Bouclet.
On the scattering theory of asymptotically flat manifolds.

15h05 – 16h00 : Dietrich Häfner.
On classical and quantum scattering for field equations on the (De Sitter) Kerr metric.

16h00 – 16h30 : Coffee, tea, cookies.

16h30 – 17h25 : Sylvain Golénia.
Few result on the asymptotic of eigenvalues for the discrete Laplacian

Conference dinner at the restaurant "Ici et Ailleurs"
(17 allée des acacias, 95000 Cergy-Pontoise)

Friday, June 24

09h30 – 10h00 : Coffee, tea, croissants

10h00 – 10h55 : Francis Nier.
PI-condition and multiscale mean-field analysis.

11h05 – 12h00 : Mathieu Lewin.
A many-body RAGE theorem

12h00 – 14h00 : Lunch.

Titles and Abstracts

Tuesday, June 21

Jacob Schach Møller (Aarhus University)

TITLE : *Local Spectral Deformation*

ABSTRACT : It is well known that the usefulness of dilation analyticity is intimately connected with the role of the generator of dilation in Mourre theory for Schrodinger operators. If one expands the dilated Hamiltonian in powers of a dilation parameter, the leading order correction to the Hamiltonian is the commutator between the Hamiltonian and the dilation generator. In principle, a Mourre estimate should therefore cause the non-normal dilated Hamiltonian to form a gap in its essential spectrum, locally where a Mourre estimate is valid. This will lay bare embedded eigenvalues and enable an application of Kato's analytic perturbation theory.
In this talk we pursue this scheme in a an abstract setting, and apply it to two-body dispersive system where we show that the non-threshold part of the embedded energy-momentum point spectrum form a semi-analytic subset of the complement of the threshold set in energy-momentum space.
The talk is based on joint work with Matthias Engelman and Morten Grud Rasmussen.

Matti Lassas (University of Helsinki)

TITLE : *Scattering in complex geometrical optics and the inverse problem for the conductivity equation*

ABSTRACT : We study Calderon's inverse problem in the two-dimensional case, that is, the question whether the properties of the conductivity function inside a domain can be determined from the voltage and current measurements made on the boundary. We determine the locations of the jumps of the conductivity function. To do this, we introduce a new method based on the propagation of singularities and the scattering of the singularities from the discontinuities of the coefficient functions. While the conductivity equation satisfied by an electrostatic field is an elliptic equation and does not propagate singularities, the associated equations which are used to construct so-called complex geometrical optics (CGO) solutions are of complex principal type. Standard hyperbolic wave equations satisfied by, e.g., acoustic waves, are examples of real principal type equations, which efficiently propagate singularities along one-dimensional characteristics, that is, along rays. Complex principal type equations propagate singularities along two-dimensional bicharacteristic leaves. In the talk we consider

the propagation and scattering of these singularities.
This is joint work with A. Greenleaf, M. Santacesaria, S. Siltanen and G. Uhlmann.

Kais Ammari (Université de Monastir)

TITLE : *Schrödinger operator on some graphs : stabilization and dispersive effects.*

ABSTRACT : In this talk we analyse the spectrum of the dissipative Schrödinger operator on binary tree-shaped networks. As applications, we study the stability of the Schrödinger system using a Riesz basis as well as the transfer function associated to the system. Moreover, we study the dispersive effects associated to the Schrödinger operator with potential on star-shaped network and to the free Schrödinger operator on a tadpole graph.

Wednesday, June 22

Clotilde Fermannian Kammerer (Université Paris Est)

TITLE : *Wigner measures and effective mass theorems.*

ABSTRACT : The dynamics of an electron in a crystal in the presence of impurities is described by a wave function that solves a semi-classical Schrödinger equation where the semi-classical parameter is the ratio between the mean spacing of the lattice and the characteristic length scale of variation of the external potential. Effective Mass Theory consists in showing that, under suitable assumptions on the initial data, the wave function can be approximated in the semi-classical limit thanks to a solution of a simpler Schrödinger equation, the effective mass equation, which is independent of the semi-classical parameter. Our goal in this talk is to describe how Wigner measure approach, conjugated with Floquet-Bloch decomposition, can be used to derive effective mass equations. We shall mainly consider two different situations depending on the geometric structure of the set of critical points of Bloch bands : when it consists of isolated points or when it is a submanifold of codimension larger than 1. These results are joint work with Victor Chabu and Fabricio Macia.

Spyros Alexakis (University of Toronto)

TITLE : *Singularity formation in Black hole interiors.*

ABSTRACT : The prediction that solutions of the Einstein equations in the interior of black holes must always terminate at a singularity was originally conceived by Penrose in 1969, under the name of "strong cosmic censorship hypothesis".

The nature of this break-down (i.e. the asymptotic properties of the space-time metric as one approaches the terminal singularity) is not predicted, and remains a very hotly debated issue to this day. One key question is the causal nature of the singularity (space-like, vs null for example). Another is the rate of blow-up of natural physical/geometric quantities at the singularity. Mutually contradicting predictions abound in this topic. Much work has been done under the assumption of spherical symmetry (for various matter models). We present recent developments (partly due to the speaker and G. Fournodavlos) which go well beyond this restrictive class. A key role is played by the axial symmetry reduction of the Einstein equations, where a wave map structure appears.

Jan Derezinski (Warsaw University)

TITLE : *Propagators on curved spacetimes*

ABSTRACT : The Klein-Gordon equation has several natural Green's functions, often called propagators. The so-called Feynman propagator, used in quantum field theory, has a clear definition on static spacetimes. I will discuss, partly on a heuristic level, its possible generalizations to the non-static case. I will also describe a curious open problem about the self-adjointness of the Klein-Gordon operator.

Victor Nistor (Université de Lorraine)

TITLE :*Cross-product algebras and essential spectra : applications to the N-body problem.*

ABSTRACT : I will begin by reminding some results due to Georgescu and others on how to use cross-product algebras and their representations to determine the essential spectrum of operators affiliated to these algebras. Then I shall use these results to determine the essential spectrum of N-body hamiltonians with asymptotically homogeneous interactions. At the end of my talk, I will briefly discuss some extensions of these results to asymptotically hyperbolic and other spaces using the related concept of a groupoid. The results of this talk are based on joint works with Georgescu, Lauter, Monthubert, and Prudhon.

Thursday, June 23

Christian Gérard (Université Paris 11)

TITLE : *Aspects of Quantum Field Theory in blackhole spacetimes.*

ABSTRACT : We will first present the algebraic formalism of free quantum Klein-Gordon fields in general curved spacetimes, emphasizing the difficulties caused

by the absence of a reference vacuum state. We will then illustrate the ambiguity of the notion of vacuum state by two emblematic effects in QFT on curved spacetimes, the *Unruh effect* and the *Hawking effect*. The *Hawking effect* leads to the notion of the *Hawking temperature* of a blackhole, related to its surface gravity through the construction of the *Hartle-Hawking state* on a blackhole spacetime. We will present a new point of view on the Hartle-Hawking state, which allows to greatly simplify its construction.

Benoit Pausader (Brown University)

TITLE : *Norm growth for the cubic nonlinear Schrodinger equation.*

ABSTRACT : This is a joint work with N. Tzvetkov. The question of whether the nonlinear cubic Schrodinger equation has solution which develop "small scale creation" (as measured as growth of high order Sobolev norm) is delicate. In many cases (e.g. in the Euclidean space, on the 1 dimensional torus...), it is not possible due to either scattering or complete integrability. However, when the domain is a "cylinder" $\mathbb{R} \times T^2$, this is possible. Another interesting fact is that there exists solutions whose norm H^s grows unboundedly for some $0 < s < 1$, despite the existence of a conservation law at $s = 0$ and $s = 1$.

Jean-Marc Bouclet (Université Toulouse 3)

TITLE : *On the scattering theory of asymptotically flat manifolds.*

ABSTRACT : I will survey both classic and recent results about the scattering theory of the Laplace-Beltrami operator on asymptotically flat manifolds. I will discuss time decay estimates, high and low frequency resolvent estimates, global smoothing estimates for the Schrödinger equation and their connections with Strichartz inequalities.

Dietrich Häfner (Université de Grenoble Alpes)

TITLE : *On classical and quantum scattering for field equations on the (De Sitter) Kerr metric.*

ABSTRACT : In this talk I will review some results concerning scattering theory for field equations on the (De Sitter) Kerr metric. The (De Sitter) Kerr metric is a solution of the vacuum Einstein equations with zero (positive in the De Sitter case) cosmological constant describing rotating black holes. There is a fundamental difference between Dirac and Klein-Gordon fields. Whereas there exists a positive conserved quantity for Dirac fields, no such quantity exists for Klein-Gordon fields. I will describe asymptotic completeness results for both Dirac and Klein-Gordon fields (Kerr case for Dirac and De Sitter Kerr case for Klein-Gordon). For Klein-Gordon fields the angular momentum of the solution has to be fixed. For Dirac fields I will also discuss a theorem describing the Hawking effect. This effect predicts the creation of particles by black holes. The results presented here were obtained in different collaborations with Vladimir Georgescu, Christian GÃľrard and Jean-Philippe Nicolas.

Sylvain Golénia (Université de Bordeaux)

TITLE : *Few result on the asymptotic of eigenvalues for the discrete Laplacian*

ABSTRACT : We review some recent results about Weyl-type asymptotic for discrete Laplacians acting on graphs.

Friday, June 24

Francis Nier (Université Paris 13)

TITLE : *PI-condition and multiscale mean-field analysis.*

ABSTRACT : Within our series of works with Z. Ammari, we identified a compactness condition, called the (PI) condition, within the bosonic mean field analysis. I will first recall this condition and its consequences and how it is violated while considering a Bose gas at finite temperature, leading to a mixture of condensate and non condensate phases. Then I will explain how multiscale analysis provides a natural description of those two phases. Such a multiscale analysis relying mainly on Wick observables, also provides tools for the fermionic case.
Work in progress with Z. Ammari and S. Breteaux

Mathieu Lewin (Université Paris Dauphine)

TITLE : *A many-body RAGE theorem*

ABSTRACT : The RAGE theorem is a famous result of Ruelle, Amrein, Georgescu and Enss about the long time dynamics of the Schrödinger equation. In this talk I will describe a new RAGE theorem for N-body quantum systems, proved in collaboration with Jonas Lampart (Paris-Dauphine). The result says that only bound states of systems with $0 \leq n \leq N$ particles persist in the long time average. The limit is formulated by means of an appropriate weak topology for many-body systems.