Aspects of Quantum Field Theory on black hole Spacetimes

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Quantum Field Theory in curved spacetimes

- describes *quantum* fields , Klein-Gordon, Dirac, Maxwell fields etc, propagating in a *classical* spacetime , described by a Lorentzian manifold (M, g).
- use: description of quantum phenomena in strong gravitational fields: cosmological models, neighborhood of a blackhole horizon.
- gravitation is treated classically: the theory cannot be a fundamental one.
- peculiarities: the notion of vacuum state becomes problematic: need for an algebraic framework (no reference Hilbert space).

Globally hyperbolic spacetimes

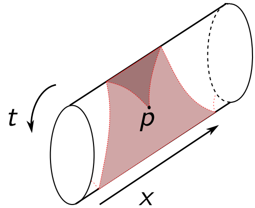
- *Lorentzian manifold*: manifold M equipped with a Lorentzian metric g, of signature $(1, d)$.
- if $v \in T_pM$, v is *spacelike, causal, timelike, lightlike* if $v \cdot g_p v > 0, \leq 0, < 0, = 0$.
- one extends this terminology to vector fields, then to piecewise C^1 curves.
- (M, g) is a *spacetime* if M is *time orientable*, i.e. there exists a continuous timelike vector field on M.
- one writes $q \in J^{\pm}(p)$ if one can join p to q by a future directed causal curve. For $K \subset M$ one sets $J^{\pm}(K) = \bigcup_{p \in K} J^{\pm}(p)$.
- a hypersurface $\Sigma \subset M$ is *Cauchy* if any inextensible causal curve in M intersects Σ at one and only one point.

Globally hyperbolic spacetimes

Definition (M, g) is *globally hyperbolic* if one of the following equivalent conditions holds:
1) M has a Cauchy hypersurface.
2) for all $p, q \in M$ $J^{+}(p) \cap J^{-}(q)$ is compact and there are no *closed* causal curves.
This definition depends only on the causal structure of (M, g).
Global hyperbolicity has important consequences for the Klein-Gordon in M:
1) the Cauchy problem is well posed,
2) there exists advanced and retarded Green's functions.

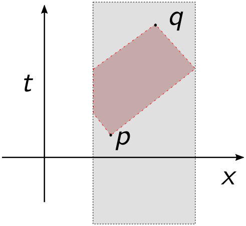
Examples of non globally hyperbolic spacetimes

$$M = \mathbb{S}^1_t \times \mathbb{R}_x$$



$J^{\pm}(p) = M$

$$M = \mathbb{R}_t \times]a, b[_x$$



$J^{+}(p) \cap J^{-}(q)$ non compact

The Klein-Gordon equation

Assume (M, g) globally hyperbolic. Let

$$P = -\nabla^a \nabla_a + m^2 = |g|^{-\frac{1}{2}} \partial_\mu g^{\mu\nu} |g|^{\frac{1}{2}} \partial_\nu + m^2,$$

the *Klein-Gordon* operator on M. (One can replace m^2 by a real C^∞ function).
P is selfadjoint for the scalar product $(u|v) = \int_M \bar{u} v d\mathrm{Vol}_g$.
Denote by $\mathrm{Sol}_{\mathrm{sc}}(KG)$ the space of C^∞ *space compact* solutions (intersection of support with a spacelike hypersurface is compact).
If Σ is a spacelike Cauchy hypersurface and $\phi_1, \phi_2 \in \mathrm{Sol}_{\mathrm{sc}}(KG)$, the quantity:

$$\bar{\phi}_1 \cdot \sigma \phi_2 := \int_\Sigma n^\mu \partial_\mu \bar{\phi}_1 \phi_2 - \bar{\phi}_1 n^\mu \partial_\mu \phi_2 dS_g$$

is independent on the choice of Σ, $(\mathrm{Sol}_{\mathrm{sc}} KG, \sigma)$ is a symplectic space.

Green's functions

Theorem There exist unique linear maps $E^{\pm}: C_0^{\infty}(M) \to C^{\infty}(M)$ such that

$$P \circ E^{\pm} = E^{\pm} \circ P = \mathbb{1},$$
$$\mathrm{supp} E^{\pm} f \subset J^{\pm}(\mathrm{supp} f), \;\; f \in C_0^{\infty}(M).$$

One has $(E^{\pm})^* = E^{\mp}$, $E := E^{+} - E^{-}$, called *Pauli-Jordan distribution* is *anti-selfadjoint*.

Green's functions

Theorem

- $\mathrm{Ran}E = \mathrm{Sol}_{\mathrm{sc}}(KG)$, $\mathrm{Ker}E = PC_0^\infty(M)$,
- $\overline{Eu_1} \cdot \sigma Eu_2 = -(u_1|Eu_2)$, $u_1, u_2 \in C_0^\infty(M)$.

One deduces that $(C_0^\infty(M)/PC_0^\infty(M), -E)$ is symplectic and

$$E : (C_0^\infty(M)/PC_0^\infty(M), -E) \to (\mathrm{Sol}_{\mathrm{sc}}(KG), \sigma)$$

is a symplectomorphism.

CCR algebras

To each $u \in C_0^\infty(M)$ we associate 'operators' aka quantum fields $\psi(u)$, $\psi^*(u)$ subject to the following rules:

- the map $C_0^\infty(M) \ni u \mapsto \psi^*(u)$ resp. $\psi(u)$ is linear, resp. *anti-linear*.
- the *canonical commutation relations* hold:

$$[\psi(u_1), \psi(u_2)] = [\psi^*(u_1), \psi^*(u_2)] = 0,$$
$$[\psi(u_1), \psi^*(u_2)] = \mathrm{i}^{-1}(u_1|Eu_2)\mathbb{1}, \ u_1, u_2 \in C_0^\infty(M),$$
$$\psi(u)^* = \psi^*(u).$$

- The $*-$algebra generated by the $\psi^{(*)}(u)$ for $u \in C_0^\infty(M)$ is denoted $\mathrm{CCR}(KG)$. Its is interpreted as the algebra of *observables* for a Klein-Gordon field.

- *locality*: if $\mathrm{supp} u_1$ and $\mathrm{supp} u_2$ are causally disjoint, then $[\psi^{(*)}(u_1), \psi^{(*)}(u_2)] = 0$.

Klein-Gordon field

- Set formally

$$\psi(u) =: \int_M \psi(x)\bar{u}(x)dVol_g,$$

since the map $u \mapsto \psi^{(*)}(u)$ should pass to the quotient by $PC_0^\infty(M)$, one should have:

$$\psi(Pu) = 0 \Rightarrow P\psi(x) = 0.$$

Hence we obtain 'operator valued solutions' of the Klein-Gordon equation.

Quasi-free states

The *states* of the quantized Klein-Gordon field are given by linear functionals on $\mathrm{CCR}(KG)$ with:

$$\omega : \mathrm{CCR}(KG) \to \mathbb{C},\ \omega(\mathbb{1}) = 1,\ \omega(A^*A) \geq 0,\ \forall A \in \mathrm{CCR}(KG).$$

A natural class of states is given by the *quasi-free states*, analogs in the non-commutative case of *gaussian measures*.

Definition

A state ω on $\mathrm{CCR}(KG)$ is *quasi-free* if:

$$\omega(\prod_1^n \psi^*(u_i) \prod_1^p \psi(v_i)) = 0,\ n \neq p,$$
$$\omega(\prod_1^n \psi^*(u_i) \prod_1^n \psi(v_i)) = \sum_{\sigma \in S_n} \prod_1^n \omega(\psi^*(u_i)\psi(v_{\sigma(i)})).$$

Quasi-free states

The quasi-free states are completely determined by their *'two-point functions'* or covariances:

$$\bar{u} \cdot \Lambda_- v := \omega(\psi^*(v)\psi(u)), \; u, v \in C_0^\infty(M),$$

- it is useful to consider also

$$\bar{u} \cdot \Lambda_+ v := \omega(\psi(u)\psi^*(v)).$$

The covariances $\Lambda_\pm$ are sesquilinear forms on $C_0^\infty(M)$,with two properties:

$$1) \quad \Lambda_+ - \Lambda_- = \mathrm{i}^{-1}E, \text{ commutation relations}$$

$$2) \quad \Lambda_\pm \geq 0, \text{ positivity.}$$

Conversely a pair of covariances $\Lambda_\pm$ such that 1) et 2) hold determines a unique quasi-free state ω.

Quasi-free states

- continuity hypothesis: one assumes that $\Lambda_\pm$ are *continuous* on $C_0^\infty(M)$:
- consequence: there exist $\Lambda_\pm \in D'(M\times M)$ such that:

$$
\begin{array}{rl}
1) & P_x\Lambda_\pm(x,y) = P_y\Lambda_\pm(x,y) = 0, \\
2_+) & \omega(\psi(u)\psi^*(v)) = \int_{M\times M}\Lambda_+(x,y)\bar{u}(x)v(y)dxdy, \\
2_-) & \omega(\psi^*(v)\psi(u)) = \int_{M\times M}\Lambda_-(x,y)\bar{u}(x)v(y)dxdy.
\end{array}
$$

One has again $\Lambda_+ - \Lambda_- = -\mathrm{i}E$, $\Lambda_\pm \geq 0$.
Important consequence of 1): $\Lambda_\pm$ are entirely determined by their restriction to an arbitrary neighborhood of $\Sigma\times\Sigma$, where $\Sigma\subset M$ is a Cauchy surface ("time-slice axiom").

Cauchy surface covariances

One can replace the symplectic space $E : (C_0^\infty(M)/PC_0^\infty(M), -E)$ by $(\mathrm{Sol}_{\mathrm{sc}}(KG), \sigma)$ or also, using the Cauchy problem by

$$(C_0^\infty(\Sigma) \oplus C_0^\infty(\Sigma), \sigma),$$

for

$$\overline{f} \cdot \sigma f = \int_\Sigma \overline{f}_1 f_0 - \overline{f}_0 f_1 ds_\Sigma, f = \rho\phi = \begin{pmatrix} \phi\restriction_\Sigma \\ \mathrm{i}^{-1}\partial_\nu \phi\restriction_\Sigma \end{pmatrix}.$$

The covariances corresponding to $\Lambda^\pm$ are denoted by $\lambda^\pm$ (2×2 matrices), called the Cauchy surface covariances.
It is convenient to introduce

$$c^\pm = \pm(\mathrm{i}\sigma)^{-1} \circ \lambda^\pm.$$

One has $c^+ + c^- = \mathbb{1}$, $c^\pm$ are projections iff the state ω is a pure state.

The Minkowski vacuum

The basic example is the *vacuum state* $\omega_{\rm vac}$ on Minkowski spacetime.

Theorem(Minkowski case) there exists a unique (pure) quasi-free state $\omega_{\rm vac}$ with the following properties:

1) $\omega_{\rm vac}$ invariant under the Poincaré group $SO(\mathbb{R}^{1,d}) \rtimes \mathbb{R}^{1+d}$.

2) $\bar{u}\Lambda_{\pm}v = \int_{\mathbb{R}^{1+d}\times\mathbb{R}^{1+d}} \bar{u}(x)\Lambda_{\pm}(x-y)v(y)dxdy$, with $\hat{\Lambda}_{\pm}(\tau, k)$ supported in $\pm\tau > 0$ (*positivity of the energy*).

One has:

$$\Lambda_{\pm}(t, \mathrm{x}) = (2\pi)^{-\mathrm{d}} \int \mathrm{e}^{\mathrm{i}(\mathrm{x}\cdot\mathrm{k}\pm \mathrm{t}\epsilon(\mathrm{k}))}\epsilon(\mathrm{k})^{-1}\mathrm{dk},$$

where $\epsilon(k) = (k^2 + m^2)^{\frac{1}{2}}$, *energy of a relativistic particle of mass m*.

What is the vacuum state good for ?

- the vacuum state provides us with
1- a Hilbert space (link with Quantum Mechanics);
2- a notion of particles (excitations of the vacuum state).
Hilbert space: GNS construction: equip $\mathrm{CCR}(KG)$ with the scalar product

$$\langle A|B\rangle := \omega_{\mathrm{vac}}(A^* B).$$

- passing to quotient and completion $\to$ a Hilbert space $\mathcal{H}$, with a *distinguished vector* $\Omega \sim \mathbb{1}$.
- The space $\mathcal{H}$ is a bosonic Fock space build on a one particle space $\mathfrak{h}$.
- Acting with field operators on the vacuum state *creates particles*: the GNS representation of $\mathrm{CCR}(KG)$ is a Fock representation.

What is the vacuum state good for ?

- Working on $\mathcal{H}$ one can:
1- rigorously construct interacting models in low dimensions: (Glimm-Jaffe 1970, $P(\varphi)_2$, φ_3^4 models).
2- formulate the perturbative renormalization:
emblematic problem : give a meaning to $\psi^*(x)\psi(x)$ (charge density): ultraviolet problem $\sim$ multiplication of distributions.
solution: Wick ordering:
one replaces $\psi^*(x)\psi(y)$ by

$$\psi^*(x)\psi(y) - \Lambda^-_{\rm vac}(x,y)\mathbb{1} = :\psi^*(x)\psi(y):.$$

The trace on $x = y$ is well defined as operator valued distribution on $\mathcal{H}$.

Hadamard states

- the above characterization of the vacuum state does not extend to general spacetimes (except stationary ones).
- one would like to find a criterion for states who look at short distances like a vacuum state.
- leads to the notion of Hadamard states, characterized by the wavefront set of their two point functions.

For $x \in M$ denote by $V^{\pm}(x) \subset T_x M$ the future/past lightcones at x.

Their dual cones $V^*_{\pm}(x) \subset T^*_x M$ are defined by:

$$V^*_{\pm}(x) = \{\xi \in T^*_x M \ : \ \xi \cdot v > 0 \ \forall v \in V^{\pm}(x), v \neq 0\}.$$

Interpretation: positive/negative energy cones.
$p(x, \xi) = \xi_\mu g^{\mu\nu}(x)\xi_\nu$ principal symbol of $P(x, D_x)$,
$\mathcal{N} = p^{-1}(\{0\})$ characteristic manifold of P

Hadamard states

$\mathcal{N}_\pm = \{(x,\xi) \in \mathcal{N} \ : \ \xi \in V^*_\pm(x)\}$, upper/lower energy shell of $\mathcal{N}$,
$\mathcal{N} = \mathcal{N}_+ \cup \mathcal{N}_-$,
For $X_i = (x_i, \xi_i)$ write $X_1 \sim X_2$ if $X_1, X_2 \in \mathcal{N}$, X_1, X_2 on the same Hamiltonian curve for p.
Definition ω is a *Hadamard state* if

$$WF(\Lambda_\pm)' \subset \{(X_1, X_2) : X_1 \sim X_2, \ X_1 \in \mathcal{N}_\pm\}.$$

- Hadamard states exist.
- their covariances are all the same modulo smooth kernels.

The Unruh effect

The *Rindler wedge* R in $\mathbb{R}^{1,1}$ is the region $\{|t| < x\}$. Equipped with the metric $-dt^2 + dx^2$ it is a spacetime.

One introduces the new coordinates

$$T = \mathrm{argth}(\frac{t}{x}), X = \ln((x^2 - t^2)^{\frac{1}{2}}) \Leftrightarrow t = \mathrm{e}^X \sinh T,\ x = \mathrm{e}^X \cosh T,$$

R becomes $\mathbb{R}_T \times \mathbb{R}_X$ with the metric:

$$ds^2 = \mathrm{e}^{2X}(-dT^2 + dX^2),$$

invariant under translations in T.

- the boost:

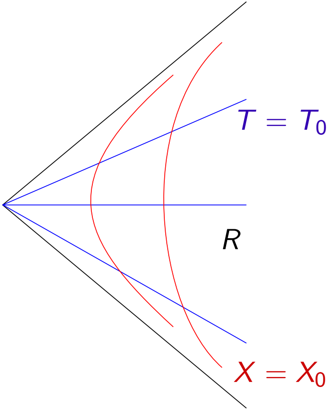
$$\alpha_s = \begin{pmatrix} \cosh s & \sinh s \\ \sinh s & \cosh s \end{pmatrix},$$

in $\mathbb{R}^{1,1}$ becomes in R the translation in T:

$\tilde{\alpha}_s : (T, X) \mapsto (T + s, X)$.

- The curve $\{\tilde{\alpha}_s(T_0, X_0)\}_{s\in\mathbb{R}}$: world line of a uniformly accelerated observer with acceleration e^{-X_0}.

The Rindler wedge



- consider the covariance of the vacuum state $\omega_{\rm vac}$:

$$\Lambda_+(t,t',x,x') = \int_{\mathbb{R}} \mathrm{e}^{\mathrm{i}F(t,t',x,x',k)}\epsilon(k)^{-1}dk,$$
$$F(t,t',x,x',k) = \epsilon(k)(t-t') + k(x-x').$$

- pass to T, T', X, X' coordinates:

$$\tilde{\Lambda}_+(T,T',X,X') = \int_{\mathbb{R}} \mathrm{e}^{\mathrm{i}\tilde{F}(T,T',X,X',k)}\epsilon(k)^{-1}dk,$$
$$\tilde{F}(T,T',X,X',k) = (\mathrm{e}^X \sinh T - \mathrm{e}^{X'} \sinh T')\epsilon(k)$$
$$+(\mathrm{e}^X \cosh T - \mathrm{e}^{X'} \cosh T')k.$$

- invariance of $\omega_{\rm vac}$ under boosts: invariance of $\tilde{F}$ under

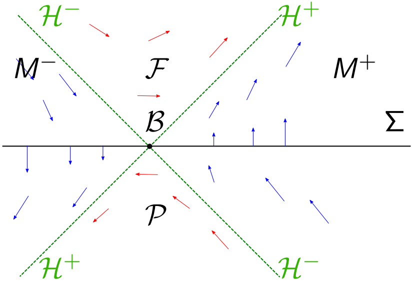
$$T \mapsto T - \frac{1}{2}(T+T'),\ T' \mapsto T' - \frac{1}{2}(T+T').$$

The Unruh effect

- conclusion: $\tilde{F}(T, T', X, X') =$
$\epsilon(k)(\mathrm{e}^{X} + \mathrm{e}^{X'})\sinh\frac{1}{2}(T - T') + k(\mathrm{e}^{X} - \mathrm{e}^{X'})\cosh\frac{1}{2}(T - T')$.
- hyperbolic trigonometry:
$\tilde{\Lambda}_{+}(T, T', X, X') = \tilde{\Lambda}_{+}(T', T + \mathrm{i}2\pi, X', X)$,
property which characterizes a *thermal state* at temperature $(2\pi)^{-1}$.
-physical interpretation: the vacuum ω_{vac} is seen by a uniformly accelerated observer with acceleration a as a *thermal state*, with temperature $a/(2\pi)$.

The Hawking effect

Consider a spacetime (M, g) describing a stationary blackhole.
Two essential features:
(M, g) admits a global, complete Killing vector field V^a.
(M, g) admits a bifurcate event horizon $\mathcal{H} = \mathcal{H}^+ \cup \mathcal{H}^-$, generated by the Killing vector field V^a.



The surface gravity

Important fact: the quantity κ defined by:

$$\kappa^2 = -\frac{1}{2}(\nabla^a V^b)(\nabla_a V_b)$$

is *constant* on $\mathcal{H}$. The constant κ is called the *surface gravity* of the blackhole.

$\mathcal{B} = \mathcal{H}^+ \cap \mathcal{H}^-$ is called the bifurcation surface, usually diffeomorphic to the sphere $\mathbb{S}^2$. V vanishes identically on $\mathcal{B}$.

Consider a free Klein-Gordon field on (M, g).

Question: does there exists a state ω *invariant* by V^a and what are its properties?

The Hartle-Hawking-Israel state

Assume that the Killing vector field V^a is *timelike* in the exterior region M^+.
Note that (M^+, g) is a globally hyperbolic spacetime, with timelike Killing vector field V^a, ie (M^+, g) is stationary.
Theorem [Kay-Wald 1991, Sanders 2013]: Assume that the Killing vector field is static in M^+. Then there exists a unique state ω_{HHI} in (M, g) with the following properties:
1) ω_{HHI} is invariant under V^a, pure in (M, g).
2) the restriction of ω_{HHI} to (M^+, g) is a *thermal* state for the group generated by V^a at Hawking temperature $T_H = \frac{\kappa}{2\pi}$.
Origin of the notion of temperature of blackholes

Killing time coordinates i nM^+

V is timelike on $\Sigma\backslash\mathcal{B}$ future directed in Σ^+, past directed in Σ^-.
Consider the right wedge M^+. It is globally hyperbolic with Cauchy surface Σ^+. Using the flow of V^a, one identifies M^+ with $\mathbb{R}\times\Sigma^+$ (Killing time) with metric

$$g = -v^2(y)dt^2 + h_{ij}(y)dy^i dy^j,$$

$v(y)$ vanishes to first order on $\mathcal{B}$.
Near $\mathcal{B}$ one can introduce Gaussian normal coordinates to $\mathcal{B}$ in (Σ, h): we identify Σ^+ with $]0,\delta[\times\mathcal{B}$, g takes the form

$$g = -v^2(s,\omega)dt^2 + ds^2 + k_{\alpha\beta}(s,\omega)d\omega^\alpha d\omega^\beta,$$

where $v(s,\omega) = \kappa s(1 + O(s^2))$.

Wedge reflection

The left wedge M^{-} is always a copy of M^{+}: there exists an involution $R : M \to M$ such that:

- R preserves g and V, reverses the time orientation;
- R maps $M^{\pm}$ onto $M^{\mp}$ and preserves Σ;
- $R = \mathrm{Id}$ on $\mathcal{B}$.

R is called a wedge reflection. It implies that $k_{\alpha\beta}(s,\omega)d\omega^{\alpha}d\omega^{\beta}$ and $v(s,\omega)$ are even , resp. odd functions of s.

The Wick rotation

Thermal states at temperature $T = \beta^{-1}$ are associated to Wick rotation, amounting to replace t by $\mathrm{i}\tau$, $\tau \in \mathbb{S}_\beta$.

- the Lorentzian manifold (M^+, g) is replaced by the Riemannian $(N^+, \hat{g})$ for

$$N^+ = \mathbb{S}_\beta \times \Sigma^+, \ \hat{g} = v^2(y)d\tau^2 + h_{ij}(y)dy^i dy^j,$$

$\mathbb{S}_\beta$ is the circle of length β.

- the Klein-Gordon operator $P = -\Box_g + m^2$ by the Laplacian $K = -\Delta_{\hat{g}} + m^2$.

The double *KMS* state

-the left wedge M^- is a copy of M^+, one can consider Klein-Gordon fields on $M^+ \cup M^-$.
- if ω_β^+ is a thermal state on M^+, one can add a 'twisted copy' of ω_β^+ on M^- and obtain a state ω_β on $M^+ \cup M^-$ called a double KMS state (Kay).
- ω_β is a pure state in $M^+ \cup M^-$.
- this construction is related to the Araki Woods representation.

Calderon projector

Let λ^+ the Cauchy surface covariance of ω_β, acting on $C_0^\infty(\Sigma^+ \cup \Sigma^-) \otimes \mathbb{C}^2$.
We map Σ^- onto Σ^+ using the wedge reflection R:

$$C_0^\infty(\Sigma^+ \cup \Sigma^-) \otimes \mathbb{C}^2 \sim C_0^\infty(\Sigma^+) \otimes \mathbb{C}^2 \oplus C_0^\infty(\Sigma^+) \otimes \mathbb{C}^2.$$

The two copies of Σ^+ are the boundary of $\Omega = [0, \beta/2]_\tau \times \Sigma^+$.
Theorem: $c^+ = (\mathrm{i}\sigma)^{-1} \circ \lambda^+$ is the Calderon projector D for $-\Delta_{\hat{g}} + m^2$ on Ω.
Proof is a tedious computation.

Calderon projector

Let $(N, \hat{g})$ a Riemannian manifold, $\Omega \subset N$ open set with smooth boundary $\partial\Omega$, $P = -\Delta_{\hat{g}} + m^2$.
-for $u \in C^\infty(\overline{\Omega})$ set

$$\gamma u = \begin{pmatrix} u\upharpoonright_{\partial\Omega} \\ \partial_\nu u\upharpoonright_{\partial\Omega} \end{pmatrix}$$

-for $f \in C^\infty(\partial\Omega) \otimes \mathbb{C}^2$ we have

$$\gamma^* f = \delta_{\partial\Omega} \otimes f_1 + \partial_\nu \delta_{\partial\Omega} \otimes f_0.$$

-the Calderon projector is the map

$$D = \gamma \circ P^{-1} \circ \gamma^*.$$

- D is a projector.
- D is a matrix of pseudodifferential operators on $\partial\Omega$.

Extension of ω_β to M

We want to extend ω_β to a state for the Klein-Gordon equation on M, i.e. extend

$$c^+ \text{ acting on } C_0^\infty(\Sigma\backslash\mathcal{B}) \otimes \mathbb{C}^2$$

to

$$c^+_{\text{ext}} \text{ acting on } C_0^\infty(\Sigma) \otimes \mathbb{C}^2.$$

We embed $\mathbb{S}_\beta \times \Sigma^+$ into $\mathbb{R}^2 \times \mathcal{B}$ as follows:

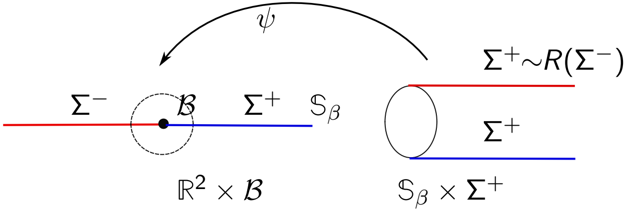
$$\psi : (\tau, s, \omega) \mapsto (s\cos(2\pi\beta^{-1}\tau), s\sin(2\pi\beta^{-1}\tau), \omega).$$

Extension of ω_β to M

The Riemannian metric $\psi^* \hat{g}$ extends as a smooth metric $\hat{g}_{\rm ext}$ on $N_{\rm ext} = \mathbb{R}^2 \times \mathcal{B}$ iff $\beta = (2\pi)/\kappa$. Then

$$T_H := \frac{\kappa}{2\pi} \text{ is the Hawking temperature}$$

For $\beta \neq (2\pi)/\kappa$, $\hat{g}_{\rm ext}$ has a conical singularity on $\{0\} \times \mathcal{B}$.



Extension of ω_β to M

- if $\beta = (2\pi)/\kappa$ then the Calderon projector $D_{\rm ext}$ acting on $C_0^\infty(\Sigma) \oplus C_0^\infty(\Sigma)$ is an extension of c^+.
- one can show that it produces a state on M, the looked for Hartle-Hawking-Israel state ω_{HHI}.
- the fact that ω_{HHI} is pure is obvious ($D_{\rm ext}$ is a projection).
- the fact that ω_{HHI} is a Hadamard state is very easy to prove, using that $D_{\rm ext}$ is pseudodifferential.

Thank you for your attention !